

MODELLING OF NONLINEARITIES WITH
APPLICATIONS TO EMD AND NEURAL
NETWORKS

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DEPARTMENT OF ELECTRICAL ENGINEERING
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Certificate

This is to certify that the thesis entitled “ **Modelling of Nonlinearities with Applications to EMD and Neural Networks**” being submitted by **Mr. Venkatappa Reddy P** to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is the record of the bona-fide research work carried out by him under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.

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New Delhi

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Abstract

In practical applications, the available data series is a nonlinear and nonstationary, nonlinear systems are used to process these data series. A linear in the parameter (LIP) model for nonlinear system is used to extend optimal linear filter theory to nonlinear filter theory. However, existing methods fail to obtain LIP models for strong nonlinear systems viz., Signum, rectified linear unit (ReLU) and an ideal comparator input-output characteristics. In this thesis, an even mirror Fourier nonlinear (EMFN) filter is used to obtain LIP models for these strong nonlinearities. We also introduce tunable ReLU as a novel nonlinear system. The characteristics of this system is similar to ReLU for larger tunable parameter values. By using strong nonlinearities, we characterize representation of empirical mode decomposition (EMD) algorithm, neural networks and deep neural networks (DNN).

A data decomposition algorithm EMD is used to decompose nonlinear and non-stationary data series. We propose a model for EMD algorithm using two strong nonlinearities and a weighted sinc interpolator that provide a mathematical tractability to represent each residual intrinsic mode function (IMF) of the data. The proposed EMD model shows better energy preservation property, orthogonality among IMFs and less computational complexity over existing method. We also discuss the application of the proposed EMD model and variational mode decomposition (VMD) algorithm for thresholding of nonuniformly illuminated images. The superiority of the proposed EMD model and VMD algorithms over other existing techniques are observed in obtaining thresholded image.

Neural networks are a class of nonlinear systems that are used to process various nonlinear data series. We present a characterization and representation of neural network architectures using tunable ReLU with reduced computational complexity. We further analyze the representation of a nonlinear block max pooling present in DNN. We observe that the linear approximation of max pooling is an average pooling. We also study the Lipschitz condition of tunable ReLU and its behavior in DNN and long short term memory (LSTM) networks. Furthermore, we study the performance of tunable ReLU in training networks on various data sets and its superiority over other nonlinear activation functions is demonstrated.

Finally, we propose models for some useful nonlinear systems such as homomorphic and median filters. We propose a model to obtain homomorphic filter like characteristics using Volterra kernels. Also, we obtain a relationship between homomorphic filter coefficients and Volterra coefficients. In the second model, we propose two new methods for an efficient modelling of median filter using threshold decomposition, a polynomial expression and a positive boolean function (PBF) based on sum. In the first method, a polynomial expression is used to obtain binary median filter characteristics. In the second method, a PBF based on sum is proposed to obtain binary median filter characteristics. The performance and computational complexities of these two models are also compared with the existing models.

सार

व्यावहारिक अनुप्रयोगों में, उपलब्ध डेटा श्रृंखला एक अरेखीय और गैर स्थिर है, इन डेटा श्रृंखला को संसाधित करने के लिए अरेखीय सिस्टम का उपयोग किया जाता है। पैरामीटर में एक रैखिक (एलआईपी) मॉडल अरेखीय प्रणाली के लिए मॉडल इष्टतम रैखिक सिद्धांत का विस्तार करने के लिए प्रयोग किया जाता है अरेखीय फिल्टर सिद्धांत। हालांकि, मौजूदा विधियां मजबूत के लिए एलआईपी मॉडल प्राप्त करने में विफल रही हैं। अरेखीय सिस्टम जैसे, साइनम, सही करनेवाला एड रैखिक इकाई (रेलु) और एक आदर्श तुलनित्र इनपुट आउटपुट विशेषताओं। इस थीसिस में, एक दर्पण फूरियर नॉनलाइनर (ईएमएफएन) इन मजबूत नॉनलीनारिटीज के लिए एलआईपी मॉडल प्राप्त करने के लिए फिल्टर का उपयोग किया जाता है। हम भी परिचय देते हैं एक उपन्यास अरेखीय प्रणाली के रूप में ट्यूनेबल रेलु। इस प्रणाली की विशेषताएं हैं बड़े ट्यूनेबल पैरामीटर मानों के लिए रीएल्यू के समान। मजबूत नॉनलीनारिटीज का उपयोग करके, हम अनुभवजन्य मोड अपघटन (ईएमडी) एल्गोरिदम के प्रतिनिधित्व की विशेषता है, तंत्रिका नेटवर्क और गहरे तंत्रिका नेटवर्क (डीएनएन)।

एक डेटा अपघटन एल्गोरिदम ईएमडी का उपयोग अरेखीय और गैर स्थिर विघटन करने के लिए किया जाता है डेटा श्रृंखला हम दो मजबूत नॉनलीनारिटीज का उपयोग कर ईएमडी एल्गोरिदम के लिए एक मॉडल का प्रस्ताव है और एक भारित सीन्स इंटरपॉलटोर जो गणितीय ट्रेक्टबिलिटी प्रदान करते हैं डेटा के प्रत्येक अवशिष्ट आंतरिक मोड फंक्शन (आईएमएफ) का प्रतिनिधित्व करने के लिए। प्रस्तावित ईएमडी मॉडल आईएमएफ के बीच बेहतर ऊर्जा संरक्षण संपत्ति, ऑर्थोगोनैलिटी दिखाता है और मौजूदा विधि पर कम कम्प्यूटेशनल जटिलता। हम आवेदन पर भी चर्चा करते हैं प्रस्तावित ईएमडी मॉडल और विविधता मोड अपघटन (वीएमडी नोन्लीफॉर्मली रोशनी छवियों के दहलीज के लिए एल्गोरिदम) की श्रेष्ठता अन्य मौजूदा तकनीकों पर प्रस्तावित ईएमडी मॉडल और वीएमडी एल्गोरिदम हैं थ्रेसहोल्ड छवि प्राप्त करने में मनाया गया। |

तंत्रिका नेटवर्क गैरलाइन प्रणाली का एक वर्ग है जिसका प्रयोग विभिन्न प्रक्रियाओं के लिए किया जाता है। अरेखीय डेटा श्रृंखला। हम तंत्रिका के एक चरित्र और प्रतिनिधित्व प्रस्तुत करते हैं। कम कम्प्यूटेशनल जटिलता के साथ ट्यूनेबल रीएल्यू का उपयोग कर नेटवर्क आर्किटेक्चर। हम आगे एक अरेखीय ब्लॉक अधिकतम पूलिंग के प्रतिनिधित्व का विश्लेषण करते हैं दन्। हम देखते हैं कि अधिकतम पूलिंग का रैखिक अनुमान एक औसत पूलिंग है। हम ट्यूनेबल रीएल्यू और डीएनएन में इसके व्यवहार की लिप्सचिट्ज की स्थिति का भी अध्ययन करते हैं लंबी शॉर्ट टर्म मेमोरी (एलएसटीएम) नेटवर्क। इसके अलावा, हम प्रदर्शन का अध्ययन करते हैं विभिन्न डेटा सेटों और इसकी श्रेष्ठता पर प्रशिक्षण नेटवर्क में ट्यूनेबल रीएल्यू का अन्य अरेखीय सक्रियण कार्यों का प्रदर्शन किया जाता है।

अंत में, हम होमोमोर्फिक जैसे कुछ उपयोगी अरेखीय सिस्टम के लिए मॉडल का प्रस्ताव और औसत लेटर। हम गुणों की तरह होमोमोर्फिक फिल्टर प्राप्त करने के लिए एक मॉडल का प्रस्ताव है वोल्टेरा कर्नेल का उपयोग कर। इसके अलावा, हम होमोमोर्फिक के बीच एक रिश्ता प्राप्त करते हैं लेटर कोएफिशिएंट्स और वोल्टेरा कोएफिशिएंट्स। दूसरे मॉडल में, हम दो नए प्रस्ताव देते हैं थ्रेसहोल्ड अपघटन का उपयोग करते हुए औसत लेटर के ई ईंट मॉडलिंग के लिए विधियां, ए बहुपद अभिव्यक्ति और योग पर आधारित सकारात्मक बूलियन फंक्शन (पीबीएफ)। में आरएसटी विधि, एक बहुपद अभिव्यक्ति का उपयोग बाइनरी मध्य लेटर विशेषताओं को प्राप्त करने के लिए किया जाता है। दूसरी विधि में, योग पर आधारित पीबीएफ बाइनरी प्राप्त करने का प्रस्ताव है औसत लेटर विशेषताओं। प्रदर्शन और कम्प्यूटेशनल जटिलताओं इन दो मॉडलों की तुलना मौजूदा मॉडलों से भी की जाती है।

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List of Abbreviations

EMD	Empirical mode decomposition
ReLU	Rectified linear unit
EMFN	Even mirror Fourier nonlinear filter
DNN	Deep neural networks
VMD	Variational mode decomposition
LSTM	Long short term memory network
PBF	Positive boolean function
LIP	Linear in the parameter
ELU	Exponential linear unit
IMF	Intrinsic mode function
MLP	Multilayer perceptron
PSNR	Peak signal to noise ratio
AMBE	Absolute mean brightness error
MSE	Mean square error