

**THERMAL ANALYSIS FOR MINIMUM FILM
THICKNESS AND FRICTION STRESS IN COLD
EXTRUSIONS OF METALS**

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Thermal Analysis for Minimum Film Thickness and Friction Stress in Cold Extrusions of Metals

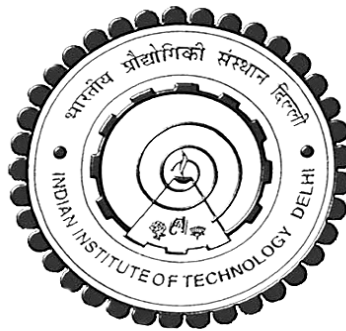
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Submitted

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to the



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Certificate

This is to certify that the thesis titled “**Thermal Analysis for Minimum Film Thickness and Friction Stress in Cold Extrusions of Metals**” being submitted by **Mr. Pankaj Tomar** (2007AMZ8333) to the **Indian Institute of Technology Delhi**, New Delhi, India, for the award of the degree of **Doctor of Philosophy** is a record of bonafide research work carried out by him under our guidance and fulfilled the requirements for the submission of this thesis. As per our awareness the results contained in this thesis have not been submitted in part or in full to any other university or institute for award of any degree or diploma.

We also certify that Mr. Pankaj Tomar has pursued the prescribed course of research at Indian Institute of Technology Delhi.

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Abstract

Cold extrusion process is employed to manufacture the products of different cross-sections. In this process, material (billet) is pushed through the dies of the desired cross-sections. The main advantage of extrusion over other manufacturing processes is its ability to produce very complex cross-sections based products even using very difficult to deform metals/materials. This happens due to the presence of compressive and shear stresses in the extrusion process. It is worth mentioning here that the rods, pipes, and frames of different cross-sections of metals are manufactured by cold extrusion processes, which are widely used in the window and door frames, roofing and exterior cladding, curtain walling, components of road and rail vehicles, marine applications etc. In cold extrusion, the presence of high friction at the interface of billet and die significantly increases the forming load and adversely influences the quality of the finished products. Therefore, effective lubrication is needed at the interface of die-billet to control the friction. Existence of proper lubricating film at the die/billet interface reduces both the interfacial friction and wear of die, which in turn efficiently produces the close tolerance and better finished products. Moreover, existence of even minor discontinuity in the flow of material during the extrusion due to improper surface profiles of die causes formation of dead metal zones, which is undesirable from product quality point of view. Therefore, the study for the maintenance of optimum friction by establishing the proper regime of lubrication (by varying the operating parameters) at the interface of die/billet is vital task for energy efficient operation and obtaining the good surface quality of the finished products.

The objectives of this thesis is to analyse the lubricated inlet and contact zones in hydrostatic and direct extrusion processes incorporating the thermal effects for the development of minimum film thickness and friction stress formulas in terms of operating parameters. Moreover, additional objective of thesis is to explore the effects of different die surface profiles (catenoidal, cycloidal, conical and exponential) on the minimum film thickness and friction stress in the extrusion processes. The parametric studies have been carried out for the wide range of operating parameters; material parameter ($G = 1.79$ to 3.23), thermal loading parameter ($L = 2.53 \times 10^{-4}$ to 6.33×10^{-3}), dimensionless extrusion pressure ($Q = 0.5$ to 1.5), extrusion ratio ($A = 1.65$ to 4.48), lubricant viscosity due to change in ambient temperature ($\eta_0 = 0.1$ to 0.3 Pa-s), Roeland's exponent ($z' = 0.5$ to 0.7), semi-die angle ($\theta = 10^0$ to 20^0) and starvation factor ($\psi = 0.1$ to 0.3). The coupled solutions of

governing equations (Reynolds equation, energy equation, film geometry expressions and rheological relations) have been achieved using finite difference approach and Lobatto quadrature technique.

Based on the simulated results reported in this thesis for the hydrostatic extrusion process using the conventional die profile (linear conical shape), it is found that the minimum film thickness in the inlet zone increases with increase in the thermal loading parameter, extrusion pressure, lubricant viscosity, extrusion ratio, Roeland's exponent, and material parameter. However, the minimum film thickness reduces with increase in the semi-die angle. The friction stress in the contact zone reduces with increase in material parameter, thermal loading parameter, extrusion pressure, and extrusion ratio. Moreover, friction stress increases with increase in Roelands exponent, viscosity of lubricating oil, and semi-die angle. Die surface profile plays substantial role in reducing the friction stress in hydrostatic extrusion process. About 70-80% reduction in friction stress is observed with catenoidal surface profile of die in comparison to conventional conical die. The trend of minimum film thickness variation in the inlet zone with catenoidal profile is similar to conventional conical profile of die. However, enhanced value of lubricating minimum film thickness is noticed with catenoidal profile of die in hydrostatic extrusion process. The analysis of die-billet interface in direct extrusion process reveals decrease in minimum film thickness in the inlet zone with increase in the material parameter, semi-die angle, and starvation factor. However, increase in the values of thermal loading parameter, Roelands exponent, and viscosity of lubricating oil enhances the minimum film thickness in the inlet zone. In direct extrusion case too, friction stress increases with increase in material parameter, semi-die angle, lubricant's viscosity, Roelands exponent, and increase in starvation factor. The catenoidal surface profile of die yields significant reduction in the friction stress that is very similar to hydrostatic extrusion case. It also reveals increase in the minimum film thickness in the inlet zone with die surface having catenoidal profile. Finally based on the simulated results incorporated in this thesis for the hydrostatic and direct extrusion processes, the empirical relations for predictions of minimum film thickness in the inlet zone and interfacial friction in contact zone as function of operating parameters have been developed for application in metallic extrusion processes.

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Nomenclature

a	friction factor
$\bar{a}$	distance specified in lubricated domain, m
a_0	constant in friction model
A	extrusion ratio
$\bar{A}$	dimensionless contact ratio
A_0	material constant
A_r	real contact area
A_s	apparent contact area
b	pressure exponent
b_0	constant in friction model
c	specific heat of billet, J/kg-K
C	thermal correction factor
C_p	specific heat of lubricant, J/kg-K
f	friction factor in real contact
G	material parameter $(\alpha \cdot \sigma_y)$
G_l	shear modulus of lubricant, MPa
h	lubricant film thickness, m
$\bar{h}$	Normalized lubricant film thickness (h / λ)
h_{full}	fully flooded film thickness of lubricant, m
h_{iso}	isothermal minimum film thickness of lubricant, m
$h_{starved}$	starved lubricated film thickness of lubricant, m
h_{th}	thermal minimum film thickness of lubricant, m

h_w	Wilson and Walowit film thickness of lubricant, $(3U_b\eta_0\alpha \exp(\alpha q) / (1 - \exp(-G)) \tan \theta)$
k_l	thermal conductivity of lubricant, W/m-K
l	characteristics length of ridge, m
L	thermal loading parameter
L'	corrected thermal loading parameter
m	Tresca friction factor (τ / τ_y)
$\bar{m}$	viscosity index
$\bar{m}_{\Delta L_n}$	Average constant friction factor
n	flow index
n_m	material constant
p	lubricant pressure, MPa
q	extrusion pressure, MPa
Q	dimensionless extrusion pressure (q / σ_y)
S	Sommerfeld number
t	Common ratio of geometric series
T	lubricant film temperature, K
$\bar{T}_k$	legendre coefficient of temperature
T_0	ambient temperature, K
u	lubricant velocity, m/s
U	surface velocity of material in work zone, m/s
U_b	billet velocity, m/s
V_0	low bound of relative velocity of billet, m/s
V_s	relative sliding speed of billet, m/s
x	co-ordinate along the extrusion direction, m
x_1	distance of inlet zone from origin, m

x_2	distance of work zone from origin, m
x_r	general distance of work zone from origin, m
y	co-ordinate perpendicular to extrusion direction, m
z	exponent in Roelands viscosity model
Z_d	average of the Zener–Hollomon parameter
+1	subscript for Lobatto location, ($\zeta = 1/\sqrt{5}$)
b	subscript for billet surface
−1	subscript for Lobatto location, ($\zeta = -1/\sqrt{5}$)
d	subscript for die surface

Greek symbols

α	pressure coefficient of viscosity, Pa ^{−1}
α_0	material constant
α_1	wedge angle, degree
$\bar{\alpha}$	real area contact ratio
β	semi-die angle in radians
θ	semi-die angle in degree
η	viscosity of lubricant, Pa-s
η_0	viscosity of lubricant at T_0 , Pa-s, Pa-s
ξ	fluidity function, $(1/\eta)$, m ² /N.s
$\bar{\xi}_k$	Legendre coefficients for fluidity
ρ	density, kg/m ³
χ	thermal diffusivity, $k_l / (\rho C_p)$
ψ	starvation factor
γ	temperature co-efficient of viscosity
$\dot{\gamma}$	shear strain
ζ	dimensionless parameter, $2y / h$

λ	die land, m
μ	coefficient of friction (τ / σ_y)
σ_y	yield strength of material, MPa
τ	friction stress, MPa
τ_0	Eyring stress, MPa
τ_a	adhesion friction stress, MPa
$\bar{\tau}_a$	average shear stress at the contacting asperity peak, MPa
$\bar{\tau}_b$	average shear stress at the lubricant pocket, MPa
τ_h	viscous friction stress, MPa
τ_p	plowing friction stress, MPa
τ_y	limiting shear stress of material, MPa