A THESIS
ON
UNSTEADY FLOW AND HEAT TRANSFER OF VISCOUS INCOMPRESSIBLE FLUIDS

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By

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CERTIFICATE

This is to certify that this thesis entitled 'Unsteady Flow and Heat Transfer of Viscous Incompressible Fluids', being submitted by Mr. R.S. Rath to the Indian Institute of Technology, New Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics is a record of bonafide research work carried out by him. Mr R.S. Rath has worked under our guidance and supervision for the last two years and has fulfilled the residential and other requirements for the submission of this thesis, which to our knowledge, has reached the requisite standard.

The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any degree or diploma.

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A C K N O W L E D G M E N T S

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C O N T E N T S

S Y N O P S I S

Problems involving unsteady flows comprise one of the most challenging research fields in fluid dynamics. The inherent mathematical difficulties in such types of problems force us to restrict analysis to simple flow configuration or to simple prescribed time variations of the flow. The Von-Kármán rotating disk problem is particularly suitable for the study of the unsteady flow problem because its velocity field is fully three dimensional, and therefore somewhat general. Also the wellknown fact that the Navier-Stokes equations for this geometry become identical to the boundary layer equations gives added importance to the study. Mention may be made of the works of Greenspan and Howard, Thiriot, Nigam, Moore, Sparrow and Gregg, Tirskii, Benton in this field. While a few were interested in the time-history of the velocity field as the body is accelerated from rest either suddenly or gradually, Moore attempted to find the deviation of the instantaneous velocity distribution from that of the instantaneous steady-state, if one such existed. The study of the rotating disk problems finds ready applications in the working of centrifugal fans, turbines and numerous other engineering devices.

While the concept of Newtonian fluid could successfully explain a number of fallacies, there still lay much to be bridged between experimental and theoretical results and for this the straightforward assumption of the fluid properties like viscosity and thermal conductivity being taken constants, is responsible in no small measure. As a matter of fact, laminar flow is of most practical interest in highly viscous liquids and in such liquids viscosity is strongly dependent on temperature. Temperature variations of the order of those encountered in practice are accompanied in some materials by a change in viscosity of 3 or more orders of magnitude. Thermal conductivity in many of the fluids are also dependent on temperature and due consideration should be given to these facts before one hopes for an agreement between theoretical and experimental investigations. Relatively few authors have given consideration to the study of flow and heat transfer with the assumption of the temperature dependence of the fluid properties. Nahme, Rosenberg and Hellums, Hays, Joseph, Tyagi are amongst the few to have worked on the above considerations.

Different constitutive equations are being proposed from time to time to take into account the non-linear effects that were ignored by the Newtonian fluid theory. Fluids with such constitutive equations can broadly be classified as non-Newtonian, the only point, common to them, being that they do not obey the linear relation between stress and rate of shear, of the Newtonian fluid. Such non-linear theories have been proposed by Noll, Rivlin, Ericksen, Oldroyd, Bingham, Eringen and others. Suspensions and slurries of rotund and plate like particles seem to be substantially devoid of shear elasticity and may be classified as simple viscous non-Newtonian fluids.

This thesis consists of seven chapters. In chapter I a brief survey of the literature on unsteady flows with particular reference to unsteadily rotating disk problems, on the flow and heat transfer in fluids with variable properties and the non-Newtonian fluids has been made. Short descriptions of the problems and important results obtained in each of the subsequent chapters have also been given.

Chapter II is a study of the unsteady flow of a viscous incompressible fluid produced by a porous disk rotating with time-dependent angular velocity $\omega(t)$, the suction velocity also being taken as a function of time. The resulting non-linear partial differential equations are solved by the Galerkin method and approximate solutions of the flow functions for different values of the parameters involved are obtained. The case of suddenly accelerated motion of the disk is deduced from the general case considered. It is found that at $\eta = 1$ (η is the non-dimensional perpendicular distance from the disk) the drag is independent of time in all cases of constant rotation of the disk irrespective of whether the suction (or influx) velocity is time-dependent or constant. In the case of time-dependent angular velocity the drag at $\eta = 1$ attains that value at $\tau = 1$ (τ is the dimensionless time).

In chapter III, we have considered the unsteady flow of a viscous incompressible fluid between two infinite disks rotating with time-dependent velocities, accompanied by time-dependent suction or/and injection. Different cases of the disks, rotating in the same sense of the opposite senses with the same speed or one of them remaining at rest, are studied. Here again, the Galerkin technique is made use of, in ~~the~~ solving the governing partial differential equations. The case of sudden acceleration of either or both of them have been considered as special cases of the time-dependent rotation. A modified Reynolds number is introduced and the growth in time of the flow functions and drag of the fluid between the disks is investigated for a particular value of the Reynolds number. It is seen that in the particular case when the disks are suddenly accelerated to the same velocity in the same direction in the absence of fluid influx and suction, the fluid mass at once rotates as a solid body with the disks. In addition to this it is also noted that when the disks are accelerated to the same rotational velocity or when one is at rest and the other is accelerated, the tangential velocity of the fluid changes linearly with the distances from the disks (and consequently the drag is constant) in conformity with the theoretical and experimental studies of Stewartson.

The fourth chapter considers natural convection flow and heat transfer, in a flow between two stationary parallel plates of a viscous incompressible fluid where viscosity and thermal conductivity are taken to be linearly varying, the former decreasing and the latter increasing, with the temperature excess of the fluid over a reference value. We assume the presence of a heat source and that the walls are maintained at constant temperatures. The cases of flow with and without suction and injection at the walls are separately considered. The temperature and velocity functions are expanded as two series in powers of a modified Brinkman number. The resulting ordinary differential equations have been solved exactly. It is found that the Nusselt numbers at the walls are independent of the variation in viscosity, whereas the skin-friction is mainly influenced by it. It is observed that with unequal wall temperatures, fluid flows slower with an increase in α_1 but faster with an increase in β_1. With no suction, the temperature of the fluid at certain places is higher than that of the hotter wall for $\alpha_1 = -.5$ and 0 but for $\alpha_1 = 1$ the fluid temperature everywhere is less than that of the hotter wall.

In the fifth chapter we have studied the flow past an infinite flat plate, taking viscous dissipation and the temperature dependence of viscosity and thermal conductivity into account. Here again we assume the presence of a strong suction through the plate. The velocities along and normal to the plate are readily derived from a stream function. Substitution of two different series in terms of the suction parameter, for the stream function and temperature function in the boundary layer equations for the momentum and heat balance give us sets of ordinary differential equations. The first third and the fifth sets have been numerically integrated (the second and fourth sets have trivial solutions) by the Runge-Kutta procedure as outlined by Gill. Step-by-step integration has been carried out in the interval (0,7) with the step length of o.l. Certain initial values had to be assumed for some of the dependent variables for starting the integration. A self iterative procedure to correct these starting values to the desired degree of accuracy has been evolved. Nusselt number and skin friction along with the stream function, velocity and temperature functions have been calculated. It is seen that an increase in either β_1 or α_1 decreses the skin-friction coefficient and increases the Nusselt number. Irrespective of the value of the suction parameter, with $R_x = Pr = 1$, the Nusselt number and the skin-friction coefficient have same values as in constant property fluid, in the absence of frictional dissipation. Effects of the Brinkman number and the Prandtl number on the heat transfer are found to be considerable.

The sixth chapter deals with the unsteady flow and heat transfer of a non-Newtonian fluid about an infinite disk rotating with time-dependent angular velocity. The technique of Sparrow and Gregg has been employed to determine the deviations of the actual instantaneous state of flow from the quasi-steady state. In solving the four sets of ordinary differential equations, Runge-Kutta-Gill method for numerical integration has been employed. The notable results here are that the heat transfer becomes quasi-steady after a longer time. Flow in a non-Newtonian fluid attains quasi-steady state earlier, but the heat transfer attains it later, as compared to a Newtonian fluid.

In the seventh chapter we have studied the problem of a one dimensional flow generated by a gradually accelerated flat plate in a simple inelastic (in shear) type of non-Newtonian fluid. Here the variable viscosity μ is a function of the three invariants of the rates of deformation i.e. $\mu = \mu(I_1, I_2, I_3)$. Owing to the one dimensional nature of the incompressible flow, I_1 and I_3 identically vanish and μ is assumed to be a linear function of the second invariant of the rate of deformation tensor. The single flow equation reduces to a set of ordinary differential equations of which the first has a non-trivial solution, the remaining equations have only trivial solutions.

The computations were carried out on the electronic digital computer I.B.M. 1620 of I.A.R.S., Pusa, New Delhi and I.B.M.7044 of I.I.T., Kanpur.