

**NUMERICAL MODELING OF PROPAGATION
AND ENERGY-DECAY MECHANISMS OF
NEAR-INERTIAL WAVES**

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**DEPARTMENT OF APPLIED MECHANICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI**

APRIL 2024

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NEAR-INERTIAL WAVES**

by

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DEPARTMENT OF APPLIED MECHANICS

Submitted
in fulfillment of the requirements of the degree of Doctor of Philosophy
to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

APRIL 2024

Certificate

This is to certify that the thesis entitled “**Numerical Modeling of Propagation and Energy-decay Mechanisms of Near-inertial Waves**”, submitted by **Mr. Siva Heramb Peddada** to the **Indian Institute of Technology Delhi** for the award of the degree of **Doctor of Philosophy** is a record of the original, bona fide research work carried out by him under my supervision. The thesis works meets the requisite standards and the candidate is worthy of consideration for the degree of Doctor of Philosophy in accordance with the regulations of the institute.

The results contained in this thesis have not been submitted in part or in full to any other university or institute for the award of any degree or diploma.

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Acknowledgements

I would like to express my heartfelt gratitude and appreciation to the many individuals who have played an important role in the completion of this PhD thesis.

Firstly, I am profoundly grateful to my supervisor Prof. Vamsi Krishna Chalamalla for his guidance and unwavering support. His expertise, patience, and dedication have been instrumental in shaping the direction of my research and the quality of this work. He has consistently been available for research discussions, and I could arrange meetings with him whenever necessary, which played a vital role in advancing the research work. His insightful questions periodically, on the results and the fundamentals, helped me to deepen my understanding upon revisiting them at a later point.

I extend my sincere thanks to my student research committee members, namely, Prof. Sawan Suman Sinha, Prof. Arghya Samanta and Prof. Sandeep Sukumaran for their insights, feedback, and the time they dedicated to reviewing and improving this research work. I thank Dr. Dheeraj Varma and Prof. Manikandan Mathur, who have contributed their expertise, shared ideas with me on various aspects of this research.

I also want to thank my teachers, my elder brother, my friends and fellow graduate students who have been a source of motivation, and support throughout this challenging journey. They initiated numerous engaging conversations that unveiled insightful perspectives on life. I also thank all other who have directly and indirectly been a part of my journey.

I am indebted to my parents for their unwavering love, encouragement, and faith in me at every stage in my life. I am grateful for their continuous support.

Siva Heramb Peddada

Abstract

Internal gravity waves are ubiquitous in the ocean and are known to play an important role in the ocean's energy budget. The pathways of energy transfer from large scale waves to small scale waves which are vulnerable to turbulence and mixing are crucial to understand their contribution to ocean mixing and internal wave spectrum. Near-inertial waves (NIWs) are a subset of internal gravity waves with the frequency close to the local Coriolis frequency. In this study, we investigate the propagation and mixed layer energy decay as a result of downward propagation of near-inertial waves through numerical simulations. First, we modeled the evolution of large-scale near-inertial waves (NIWs) on a β -plane by imposing initial zonal velocity in the mixed layer. We consider various background stratification values and initial velocity magnitudes, to investigate their effect on the wave characteristics and the decay rate of mixed layer kinetic energy. Increasing the background stratification strength below the mixed layer led to increased energy content in the higher vertical modes and also resulted in a faster decay of mixed-layer kinetic energy. Multiple superharmonics of near-inertial waves are observed in all the simulations, and the energy content in these superharmonics was found to increase with an increase in the interior stratification strength as well as the zonal velocity amplitude. The mixed layer energy decay rate obtained from our simulations agrees well with the theoretical estimate by [1] at lower velocity amplitudes, however, we found a significant disagreement at higher velocity amplitudes. The faster energy decay rate at higher velocity amplitudes and smaller contribution from the downward energy flux at the mixed layer base suggest larger viscous dissipation within the mixed layer. A strong shear zone is observed at the base of the mixed layer and the spatial extent of this zone was found to increase with an increase in the background stratification strength below the mixed layer and initial velocity amplitude. When we consider a non-uniform stratification below the mixed layer, we found that the intensity of the vertical shear and the spatial extent of the strong shear zone increases with an increase in the strength of the pycnocline.

In the second part of our study, we investigated the triadic resonance interactions (TRI) of freely propagating near-inertial waves in a uniformly stratified medium. We force multiple combinations of normal modes at primary frequency ω_0 from the

left boundary of the computational domain and study the spatiotemporal evolution of the wave field. We performed two sets of simulations, namely, resonant and off-resonant. For the resonant simulations, the ratio of mode numbers $m/n = 2/3$ at frequency ω_0 form a resonant triad with the superharmonic mode ($|m - n|$) having frequency $2\omega_0$. In the off-resonant simulations the primary wave frequency is chosen such that the modes with ratio $m/n = 2/3$ do not form a resonant triad. Both the resonant and off-resonant simulations show distinct peaks of energy at multiple superharmonics ($2\omega_0, 3\omega_0$, and $4\omega_0$) close to the forcing region. Away from the forcing region, these distinct peaks in the resonant simulations weaken and redistribute the energy into a broadband spectra in a length of about 3.25 times the low mode wavelength (λ_1). Whereas in the off-resonant simulations, the distinct superharmonic peaks are sustained longer and redistribute into a continuous spectrum after a length of about $7.5\lambda_1$. The spatial evolution of modal amplitudes shows that higher modes of primary wave decay faster and the corresponding higher modes of superharmonic waves grow faster. The off-resonant simulations also show significant energy in sub-harmonic frequencies, and we see a buildup of energy in the near-inertial frequency regime far from the forcing region. The frequency and the wavenumber spectra for resonant simulations reveal a -2 power law consistent with the Garrett-Munk spectrum ($E(\omega, m) \propto \omega^{-2}m^{-2}$). Through our findings, it seems that forcing multiple modes at the near-inertial frequency can lead to a GM-like energy spectrum, through non-linear wave-wave interactions. This is in contrast to the previous study [2] where they found that both inertial and M_2 tidal waves are needed to generate the GM-spectrum. Another study by Chen et al. [3] showed that M_2 tides alone can generate the GM-spectrum.

In the last part of our study, we considered modal interactions in a non-uniformly stratified medium. We found that a single mode n at frequency ω_0 undergoes self-interaction and can generate superharmonic modes $m \leq n$ at frequency $2\omega_0$ consistent with the previous study by Varma and Mathur [4]. For example, in our study, we observe that a single mode 3 at frequency ω_0 undergoes self-interaction and generates a combination of modes 1 and 2 at superharmonic frequency $2\omega_0$. The frequency spectrum show that there is a strong nonlinear interaction in the pycnocline region where there is a sharp gradient in the background density. In the case of single-mode interaction, distinct ω_0 and $2\omega_0$ peaks are observed along the length of the domain. The strength of $2\omega_0$ frequency is observed to be strongest near

the pycnocline. When multiple modes are forced, distinct ω_0 and $2\omega_0$ peaks are observed close to the forcing region. Away from the forcing region, several intermediate frequencies between ω_0 and $3\omega_0$ appear, indicating multiple modal interactions.

सार

आंतरिक गुरुत्व तरंगें समुद्र में सर्वव्यापी हैं और समुद्र के ऊर्जा बजट में महत्वपूर्ण भूमिका निभाने के लिए जानी जाती हैं। बड़े पैमाने की तरंगों से छोटे पैमाने की तरंगों तक ऊर्जा हस्तांतरण के मार्ग जो अशांति और मिश्रण के प्रति संवेदनशील होते हैं, समुद्र के मिश्रण और आंतरिक तरंग स्पेक्ट्रम में उनके योगदान को समझने के लिए महत्वपूर्ण हैं। निकट-जड़त्वीय तरंगें (NIWs) आंतरिक गुरुत्वाकर्षण तरंगों का एक उपसमूह हैं जिनकी आवृत्ति स्थानीय कोरिओलिस आवृत्ति के करीब होती है। इस अध्ययन में, हम संख्यात्मक सिमुलेशन के माध्यम से निकट-जड़त्वीय तरंगों के नीचे की ओर प्रसार के परिणामस्वरूप प्रसार और मिश्रित परत ऊर्जा क्षय की जांच करते हैं। सबसे पहले, हमने मिश्रित परत में प्रारंभिक जोनल वेग लगाकर β -प्लेन पर बड़े पैमाने पर NIWs के विकास का मॉडल तैयार किया। हम तरंग विशेषताओं और मिश्रित परत गतिज ऊर्जा की क्षय दर पर उनके प्रभाव की जांच करने के लिए विभिन्न पृष्ठभूमि स्तरीकरण मूल्यों और प्रारंभिक वेग परिमाणों पर विचार करते हैं। मिश्रित परत के नीचे पृष्ठभूमि स्तरीकरण शक्ति को बढ़ाने से उच्च ऊर्ध्वाधर मोड में ऊर्जा सामग्री में वृद्धि हुई और इसके परिणामस्वरूप मिश्रित परत गतिज ऊर्जा का तेजी से क्षय हुआ। सभी सिमुलेशन में निकट-जड़त्वीय तरंगों के कई सुपरहार्मोनिक्स देखे गए हैं, और इन सुपरहार्मोनिक्स में ऊर्जा सामग्री आंतरिक स्तरीकरण शक्ति के साथ-साथ जोनल वेग आयाम में वृद्धि के साथ बढ़ती पाई गई थी। हमारे सिमुलेशन से प्राप्त मिश्रित परत ऊर्जा क्षय दर कम वेग आयामों पर (मोहेलिस ओर स्मिथ 2001) के सैद्धांतिक अनुमान से अच्छी तरह सहमत है, हालांकि, हमें उच्च वेग आयामों पर एक महत्वपूर्ण असहमति मिली। उच्च वेग वाले आयामों पर तेज़ ऊर्जा क्षय दर और मिश्रित परत के आधार पर नीचे की ओर ऊर्जा प्रवाह से छोटा योगदान मिश्रित परत के भीतर बड़े चिपचिपापन अपव्यय का संकेत देता है। मिश्रित परत के आधार पर एक मजबूत कतरनी क्षेत्र देखा जाता है और इस क्षेत्र की स्थानिक सीमा पृष्ठभूमि स्तरीकरण शक्ति मिश्रित परत के नीचे और प्रारंभिक वेग आयाम में वृद्धि के साथ बढ़ती पाई गई। जब हम मिश्रित परत के नीचे एक गैर-समान स्तरीकरण पर विचार करते

हैं, तो हमने पाया कि ऊर्ध्वधर कतरनी की तीव्रता और मजबूत कतरनी क्षेत्र की स्थानिक सीमा पाइक्नोक्लाइन की ताकत में वृद्धि के साथ बढ़ जाती है।

हमारे अध्ययन के दूसरे भाग में, हमने एक समान रूप से स्तरीकृत माध्यम में निकट-जड़त्वीय तरंगों के स्वतंत्र रूप से प्रसार के त्रियादिक अनुनाद इंटरैक्शन (TRI) की जांच की। हम कम्प्यूटेशनल डोमेन की बाईं सीमा से प्राथमिक आवृत्ति ω_0 पर सामान्य मोड के कई संयोजनों को मजबूर करते हैं और तरंग क्षेत्र के स्पोटियोटेम्पोरल विकास का अध्ययन करते हैं। हमने सिमुलेशन के दो सेट, अर्थात् अनुनाद और ऑफ-प्रतिध्वनि का संचालित किया। गुंजयमान सिमुलेशन के लिए, आवृत्ति ω_0 पर मोड संख्या $m/n=2/3$ का अनुपात सुपरहार्मोनिक मोड ($|m-n|$) के साथ एक गुंजयमान त्रय बनाता है जिसकी आवृत्ति $2\omega_0$ होती है। ऑफ-रेजोनेंट सिमुलेशन में प्राथमिक तरंग आवृत्ति को इस तरह चुना जाता है कि $m/n=2/3$ अनुपात वाले मोड एक अनुनाद त्रय नहीं बनाते हैं। रेजोनेंट और ऑफ-रेजोनेंट दोनों सिमुलेशन फोर्सिंग क्षेत्र के करीब कई सुपरहार्मोनिक्स ($2\omega_0, 3\omega_0, 4\omega_0$) पर ऊर्जा के अलग-अलग शिखर दिखाते हैं। फोर्सिंग क्षेत्र से दूर, गुंजयमान सिमुलेशन में ये अलग-अलग चोटियाँ कमजोर हो जाती हैं और ऊर्जा को ब्रॉडबैंड स्पेक्ट्रा में कम मोड तरंग दैर्ध्य (λ_1) से लगभग 3.25 गुना की लंबाई में पुनर्वितरित करती हैं। जबकि ऑफ-रेजोनेंट सिमुलेशन में, अलग-अलग सुपरहार्मोनिक शिखर लंबे समय तक बने रहते हैं और लगभग $7.5\lambda_1$ की लंबाई के बाद एक सतत स्पेक्ट्रम में पुनर्वितरित होते हैं। मोडल आयामों के स्थानिक विकास से पता चलता है कि प्राथमिक तरंग के उच्च मोड तेजी से क्षय होते हैं और सुपरहार्मोनिक तरंगों के संबंधित उच्च मोड तेजी से बढ़ते हैं। ऑफ-रेजोनेंट सिमुलेशन भी उप-हार्मोनिक आवृत्तियों में महत्वपूर्ण ऊर्जा दिखाते हैं, और हम ऊर्जा का एक निर्माण देखते हैं फोर्सिंग क्षेत्र से दूर निकट-जड़त्वीय आवृत्ति शासन। गुंजयमान सिमुलेशन के लिए आवृत्ति और वेवनंबर स्पेक्ट्रा गैरेट-मंक स्पेक्ट्रम ($E(\omega, m) \propto \omega^{-2} m^{-2}$) के अनुरूप -2 पावर सिद्धांत को प्रकट करते हैं। हमारे निष्कर्षों के माध्यम से, ऐसा लगता है कि निकट-जड़त्वीय आवृत्ति पर कई मोड को मजबूर करने से गैर-रेखीय तरंग-तरंग इंटरैक्शन के माध्यम से GM-जैसी ऊर्जा स्पेक्ट्रम हो

सकता है। यह पिछले अध्ययन सुगियामा (2009) के विपरीत है जहां उन्होंने पाया कि GM-स्पेक्ट्रम उत्पन्न करने के लिए जड़त्वीय और M_2 ज्वारीय तरंगों दोनों की आवश्यकता होती है। चैन (2019) के एक अन्य अध्ययन से पता चला है कि M_2 ज्वार केवल GM-स्पेक्ट्रम उत्पन्न कर सकता है।

हमारे अध्ययन के अंतिम भाग में, हमने एक गैर-समान रूप से स्तरीकृत माध्यम में मोडल इंटरैक्शन पर विचार किया। हमने पाया कि ω_0 आवृत्ति पर एक एकल मोड n स्व-इंटरैक्शन से गुजरता है और, वर्मा ओर माथुर (2017) के पिछले अध्ययन के अनुरूप आवृत्ति $2\omega_0$ पर सुपरहार्मोनिक मोड $m < n$ उत्पन्न कर सकता है। उदाहरण के लिए, हमारे अध्ययन में, हम देखते हैं कि आवृत्ति ω_0 पर एक एकल मोड 3 स्व-अंतःक्रिया से गुजरता है और सुपरहार्मोनिक आवृत्ति $2\omega_0$ पर मोड 1 और 2 का संयोजन उत्पन्न करता है। आवृत्ति स्पेक्ट्रम से पता चलता है कि पाइकनोक्लाइन क्षेत्र में एक मजबूत गैर-रेखीय इंटरैक्शन है जहां पृष्ठभूमि घनत्व में एक तेज ढाल है। सिंगल-मोड इंटरैक्शन के मामले में, डोमेन की लंबाई के साथ अलग-अलग ω_0 और $2\omega_0$ शिखर देखे जाते हैं। $2\omega_0$ आवृत्ति की ताकत पाइकनोक्लाइन के पास सबसे मजबूत देखी गई है। जब कई मोड को मजबूर किया जाता है, तो फोर्सिंग क्षेत्र के करीब अलग-अलग ω_0 और $2\omega_0$ शिखर देखे जाते हैं। फोर्सिंग क्षेत्र से दूर, ω_0 और $3\omega_0$ के बीच कई मध्यवर्ती आवृत्तियाँ दिखाई देती हैं, जो कई मोडल इंटरैक्शन का संकेत देती हैं।

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Abbreviations

NIWs	N ear I nertial W aves
TRI	T riadic R esonant I nteraction
PSI	P arametric S ubharmonic I nstability
WTT	W eak T urbulence T heory
FFT	F ast F ourier T ransform
ROMS	R egional O cean M odeling S oftware
POD	P roper O rthogonal D ecomposition
KPP	K P rofile P arametrization
SOMAR	S tratified O cean M odel with A daptive R efinement
PSD	P ower S pectral D ensity

Symbols

N	Buoyancy frequency
N_{max}	Maximum buoyancy frequency
g	Acceleration due to gravity
ρ	Fluid density
ρ_0	Reference density
$\bar{\rho}$	Mean background density
ρ'	Fluctuation density
x	Zonal coordinate
y	Meridional coordinate
z	Vertical coordinate
u	Zonal velocity
v	Meridional velocity
w	Vertical velocity
t	Time
ω	Wave frequency
λ_n	Wavelength of mode n
f	Coriolis Frequency
\vec{u}	Velocity vector
$\vec{\Omega}$	Rotation rate of the Earth
μ	dynamic viscosity
κ	diffusivity
α	thermal expansion coefficient
∇	Gradient
∇^2	Laplacian
k	Horizontal wavenumber
m	Vertical wavenumber

Symbols

c_p	Phase velocity
c_g	Group velocity
θ	Angle between phase velocity and horizontal
η	Wave field
ϕ	Latitude
R	Radius of the Earth
β	Meridional derivative of Coriolis frequency
c	Decay coefficient
p	Pressure
Ri_g	Gradient Richardson number
L_x	Zonal domain length
L_y	Meridional domain length
L_z	Depth
f_0	Coriolis Frequency at mid-domain
u_0	Initial zonal velocity magnitude
H_{mix}	Mixed layer depth
N_∞	Background stratification
N_p	Pycnocline strength
τ_x	Wind stress in zonal direction
τ_y	Wind stress in meridional direction
i	iota
\tilde{U}	Characteristic velocity
p'	Pressure perturbation
w'	Vertical velocity perturbation
T'	Temperature perturbation
\vec{A}	Area vector
EF	Energy Flux
F	Time-integrated energy flux
∇x	Zonal grid spacing
∇y	Meridional grid spacing
HKE	Mixed layer horizontal kinetic energy
q	Flow variable
ψ	Vertical normal mode
k_n	Horizontal wavenumber of mode n
a_j	Temporal POD coefficient of mode j

Symbols

R	Covariance matrix
S	Vertical Shear
\vec{k}_1	Horizontal wavenumber vector
$\vec{\omega}$	Frequency vector
C_{py}	Meridional phase speed
T_{50}	Time for 50% energy decay
σ_x	Parabolic ramp function
A	Amplitude
b_{bg}	Buoyancy of the background medium
W	Domain width
H	Domain height
L	Domain length
Re_f	Forcing Reynolds number
Ro_f	Forcing Rossby number
ν	Dynamic viscosity
N_x	Number of grid points in zonal direction
N_x	Number of grid points along depth
Γ	Horizontal displacement
γ_{0nR}	Real component of the modal coefficients
γ_{0nI}	Imaginary component of the modal coefficients
J	Number of frequencies in the polychromatic wave field
\hat{u}	Fourier-transformed zonal velocity
\hat{v}	Fourier-transformed meridional velocity
\hat{w}	Fourier-transformed vertical velocity
BiS	Bispectrum
z_c	Maximum buoyancy frequency depth
σ	Half-width of pycnocline