

SCHEDULING WITH OUTLIERS TO MINIMIZE LOAD AND FLOW-TIME

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SCHEDULING WITH OUTLIERS TO MINIMIZE LOAD AND FLOW-TIME

by

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Certificate

This is to certify that the thesis titled “**Scheduling with Outliers to Minimize Load and Flow-time**” being submitted by **Syamantak Das** to the **Indian Institute of Technology Delhi**, for the award of the degree of **Doctor of Philosophy**, is a record of bonafide research carried out by him under our supervision. The results obtained in this thesis have not been submitted to any other university or institute for the award of any other degree or diploma.

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Abstract

In this thesis, we consider problems of scheduling jobs in a multiprocessor environment in order to minimize some standard metrics. Central to all the problems considered is a model where one is allowed to reject or not process some jobs, but only upto a bounded fraction of the entire input. We consider such problems in both online and offline settings.

In the online setting, we work in both restricted assignments and unrelated machines models and consider the objectives of load balancing, minimizing maximum weighted flow-time and more generally, minimizing ℓ_p -norm of weighted flow-time. Online algorithms are usually analyzed using the notion of competitive ratio which compares the solution obtained by the algorithm to that obtained by an online adversary for the worst possible input sequence. Often this measure turns out to be too pessimistic, and one popular approach especially for scheduling problems has been that of “resource augmentation” which was first proposed by Kalyanasundaram and Pruhs [27]. Although resource augmentation has been very successful in dealing with a variety of objective functions, for the problems stated above, even a (arbitrary) constant speedup cannot lead to a constant competitive algorithm. In this work, we propose a new “rejection model” which requires no resource augmentation but which permits the online algorithm to not serve an epsilon-fraction of the

requests, whereas the offline optimal has to serve all of them. Under this model, we give the following results:

1. Load Balancing

For the load balancing problem where the objective is to minimize the maximum load on any machine, we give $O(\log^2 1/\varepsilon)$ -competitive algorithm which rejects at most an ε -fraction of the jobs.

2. Maximum Weighted Flow-time

- *For minimizing maximum weighted flow-time in the restricted assignments model, we give an immediate dispatch non-migratory $O(1/\varepsilon^4)$ -competitive algorithm which can reject at most an ε -fraction of the jobs by weight. We extend this result to a more general setting where the weights of a job for measuring its weighted flow-time and its contribution towards total allowed rejection weight are different. This is useful, for instance, when we consider the objective of minimizing the maximum stretch. We obtain an $O(1/\varepsilon^6)$ -competitive algorithm in this case.*
- *In the unrelated machines model, we give a $1/\varepsilon^{O(1)}$ -competitive algorithm for minimizing maximum unweighted flow-time and load balancing problems under ε -rejection.*

These results were obtained jointly with Anamitra Roy Choudhury, Naveen Garg and Amit Kumar [17].

3. ℓ_p -norm of Weighted Flow-time

For minimizing ℓ_p -norm of weighted flow-time in the restricted assignments setting, our main result is an immediate dispatch non-migratory $1/\varepsilon^{O(1)}$ -competitive algorithm when one is allowed to reject at most ε -fraction of the total weight of jobs arriving. This is in contrast with the speed augmentation model under which no online algorithm for this problem can achieve a competitive ratio independent of p .

This result was obtained jointly with Anamitra Roy Choudhury and Amit Kumar [18].

In the offline setting, we give the first logarithmic approximation for minimizing average flow-time of jobs in the restricted assignments setting under a single knapsack constraint. In a knapsack constraint setting, each job has a profit. A feasible schedule is one which processes a set of jobs with total profit of at least a quantity Π . This result is an extension of the work of Gupta, Krishnaswamy, Kumar and Segev [24] who considered the special case where the profit of each job is unity, in the identical machines model. Our algorithm is based on iteratively rounding a natural LP relaxation for this problem. This result was obtained through joint work with Suman K. Bera and Amit Kumar [13].

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