

NONLOCAL VARIATIONAL THEORY AND APPLICATIONS TO OPTIMAL CONTROL

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CERTIFICATE

This is to certify that the dissertation, Non-local Variational Theory and Applications to Optimal Control, which is being submitted by Vijay P. Bhatkar for the award of degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

This dissertation has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this dissertation have not been submitted to any other University or Institute for the award of any degree or diploma.



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