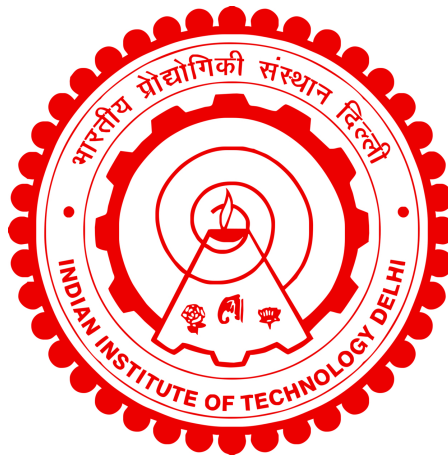


A MULTI-LEVEL FRAMEWORK FOR INTELLIGENT CONTROL OF DYNAMICAL SYSTEMS WITH UNKNOWN PHYSICS

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INDIAN INSTITUTE OF TECHNOLOGY DELHI
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A multi-level framework for intelligent control of dynamical systems with unknown physics

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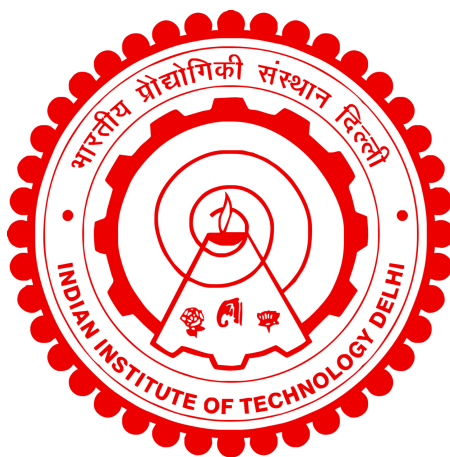
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“To every presence, natural and beyond, that guided, supported, and inspired me through this journey – your influence will remain with me always.”

Finally, I dedicate this thesis ...

To my parents, whose quiet sacrifices, endless patience, and boundless love have been the guiding light throughout my journey.

Certificate

This is to certify that the thesis titled **A multi-level framework for intelligent control of dynamical systems with unknown physics**, submitted by **Mr. Tapas Tripura**, to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by him under my supervision and guidance. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Date: May 26, 2026
Place: New Delhi

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Sincerely, Tapas Tripura

Abstract

Modern engineering systems are designed to achieve high performance in safety, stability, and efficiency, yet their operation is often challenged by uncertainties arising from environmental disturbances and varying operating conditions, necessitating the use of active control strategies. By integrating sensing, computation, and actuation, active control enables adaptive and reliable system response in real time. Recent advances in machine learning have introduced data-driven control approaches that allow real-time decision-making without explicit online optimization. However, these methods are inherently limited by a trade-off between sample efficiency and modeling bias: approaches that reduce modeling bias often require extensive interaction with the environment, while more sample-efficient methods rely on approximate models that may lack accuracy and generalizability. These challenges highlight the need for frameworks that enable reliable predictive control in partially or fully unknown environments while remaining computationally efficient. Addressing this gap, this thesis develops a suite of novel machine learning methodologies for learning parsimonious and generalizable models, enabling rapid and reliable prediction of complex nonlinear dynamical systems under parametric uncertainty, and facilitating efficient computation of optimal control policies for interaction-free, model-agnostic, predictive, and scalable decision-making.

The central challenge in model-agnostic predictive control lies in the limited availability of accurate physical knowledge, without which reliable planning of control actions becomes difficult. Existing data-driven approaches typically rely on black-box surrogate models, which often suffer from poor generalization beyond the training regime. Recent advances have addressed this limitation by discovering interpretable governing equations, yielding models that are physically consistent and capable of generalizing over long prediction horizons. Building on this progress, this thesis proposes an interpretable Lagrangian discovery framework, where Lagrangian descriptions are identified directly from data in an explicit and structured form. By integrating the resulting models with classical Model Predictive Control (MPC), a model-agnostic MPC framework is developed that enables predictive control in partially or fully unknown environments. This approach eliminates the need for continual online model updates, rendering MPC both practical and effective for complex real-world systems.

While the proposed model-agnostic data-driven MPC framework eliminates the need for repeated online model updates, its real-time deployment remains constrained by the computational cost of repeatedly simulating the discovered physics using high-fidelity solvers. This lim-

itation is particularly significant in predictive control, where system evolution must be evaluated over a finite horizon at each control step. To address this bottleneck, this thesis introduces the Wavelet Neural Operator (WNO), an operator learning framework that learns a parameterized mapping between function spaces, enabling rapid inference of complex spatiotemporal dynamics. By leveraging the timefrequency localization properties of wavelets, WNO effectively captures evolving spatial features. Furthermore, by combining wavelet-domain convolution with an integral formulation inspired by Greens theorem, WNO learns an implicit Greens kernel which, coupled with nonlinear activations, provides an efficient and accurate approximation of the underlying nonlinear operators.

Building on WNO, this thesis introduces the Multi-Fidelity Wavelet Neural Operator (MF-WNO) and the Neural Combinatorial Wavelet Neural Operator (NCWNO) to address data efficiency and generalization challenges in operator learning. MF-WNO exploits the correlation between expensive high-fidelity data and inexpensive low-fidelity data (e.g., from reduced-order models or coarse discretizations) to learn accurate high-fidelity solution operators. In parallel, NCWNO reduces data requirements by learning a combinatorial operator across multiple solution mappings and rapidly adapting to new physical systems with minimal additional data, thereby functioning as a foundation-like neural operator capable of generalizing across families of partial differential equations. By replacing repeated evaluations of computationally expensive solvers with WNO-based surrogates, the proposed framework enables accelerated planning, efficient adaptation, and continual learning without retraining from scratch, establishing a scalable and computationally efficient paradigm for surrogate-based predictive control.

Although neural-operator-enhanced model-agnostic MPC improves efficiency over classical MPC, real-time deployment remains constrained by the cost of online optimization. Deep reinforcement learning (DRL) offers an alternative via parameterized policies executable at inference; however, purely data-driven DRL is sample-intensive, while model-based variants suffer from inaccuracies due to online model learning. To address these limitations, this thesis introduces the Model-Agnostic Predictive Temporal Difference (MAP-TD) framework, which leverages a pre-trained, interpretable, and parsimonious dynamics model discovered offline using sparse Bayesian learning. This enables accurate long-horizon prediction and efficient policy learning via temporal-difference optimization, augmented with a terminal value function. Additionally, a hybrid simulation strategy combining high-fidelity solvers and a WNO-based surrogate is employed to accelerate training, resulting in interaction-free and sample-efficient control.

Finally, the feasibility of the proposed MAP-TD framework is demonstrated on two large-scale benchmark problems: wind-induced vibration mitigation in a flexible skyscraper and flow-induced vibration control in flow past a cylinder. In both cases, MAP-TD learns control policies that can be directly deployed without retraining, enabling real-time control. By leveraging a parsimonious dynamics model with sustained predictive capability, the framework eliminates the need for online model updates and avoids direct interaction with the environment during training, resulting in significantly improved sample efficiency. Quantitatively, MAP-TD achieves approximately 3000 times faster inference compared to classical MPC, while maintaining comparable or improved control performance, highlighting its effectiveness for efficient control in high-dimensional systems with unknown dynamics.

सारांश

आधुनिक इंजीनियरिंग प्रणालियों को सुरक्षा, स्थिरता और दक्षता में उच्च प्रदर्शन प्राप्त करने के लिए अभिकल्पित किया जाता है, किन्तु उनका संचालन प्रायः पर्यावरणीय व्यवधानों तथा परिवर्तनीय परिचालन परिस्थितियों से उत्पन्न अनिश्चितताओं से प्रभावित होता है, जिसके कारण सक्रिय नियंत्रण रणनीतियों की आवश्यकता उत्पन्न होती है। सेंसरिंग, संगणना और क्रियान्वयन (एक्ज्यूशन) के एकीकरण के माध्यम से, सक्रिय नियंत्रण वास्तविक समय में प्रणालियों को अनुकूली तथा विश्वसनीय प्रतिक्रिया देने में सक्षम बनाता है। मशीन लर्निंग में हालिया प्रगति ने डेटा-चालित नियंत्रण विधियों को जन्म दिया है, जो स्पष्ट ऑनलाइन अनुकूलन के बिना वास्तविक-समय निर्णय-निर्माण को संभव बनाती हैं। तथापि, ये विधियाँ स्वाभाविक रूप से नमूना-दक्षता (sample efficiency) और मॉडलिंग पूर्वाग्रह (modeling bias) के बीच एक समझौते से सीमित रहती हैं: वे दृष्टिकोण जो मॉडलिंग पूर्वाग्रह को कम करते हैं, प्रायः पर्यावरण के साथ व्यापक अंतःक्रिया की मांग करते हैं, जबकि अधिक नमूना-दक्ष विधियाँ ऐसे अनुमानित मॉडलों पर निर्भर करती हैं जिनमें सटीकता और सामान्यीकरण की कमी हो सकती है। ये चुनौतियाँ ऐसे ढाँचों की आवश्यकता को रेखांकित करती हैं जो आंशिक रूप से या पूर्णतः अज्ञात परिवेशों में विश्वसनीय भविष्यवाणी नियंत्रण को संभव बनाते हुए संगणनात्मक रूप से भी दक्ष बने रहें। इस अंतर को संबोधित करते हुए, यह शोधप्रबंध नवीन मशीन लर्निंग पद्धतियों का एक समूह विकसित करता है, जो संक्षिप्त (parsimonious) तथा सामान्यीकरण योग्य मॉडलों के अधिगम में सक्षम हैं। ये पद्धतियाँ प्राचलिक अनिश्चितता के अंतर्गत जटिल नॉनलाइनियर गतिकीय प्रणालियों की तीव्र और विश्वसनीय भविष्यवाणी को संभव बनाती हैं तथा इष्टतम नियंत्रण नीतियों की दक्ष गणना को सुगम करती हैं, जिससे अंतःक्रिया-रहित, मॉडल-निरपेक्ष, भविष्यवाणी-आधारित और मापनीय निर्णय-निर्माण संभव हो पाता है।

मॉडल-निरपेक्ष भविष्यवाणी नियंत्रण (model-agnostic predictive control) में केंद्रीय चुनौती सटीक भौतिक ज्ञान की सीमित उपलब्धता में निहित है, जिसके अभाव में नियंत्रण क्रियाओं की विश्वसनीय योजना बनाना कठिन हो जाता है। मौजूदा डेटा-चालित दृष्टिकोण सामान्यतः ब्लैक-बॉक्स सरोगेट मॉडलों पर निर्भर करते हैं, जो प्रायः प्रशिक्षण क्षेत्र से परे कमजोर सामान्यीकरण की समस्या से ग्रस्त रहते हैं। हालिया प्रगति ने इस सीमा को व्याख्येय शासनकारी समीकरणों की खोज के माध्यम से संबोधित किया है, जिससे ऐसे मॉडल प्राप्त हुए हैं जो भौतिक रूप से सुसंगत हैं और दीर्घकालिक भविष्यवाणी क्षितिजों पर सामान्यीकरण करने में सक्षम हैं। इसी प्रगति को आगे बढ़ाते हुए, यह शोधप्रबंध एक व्याख्येय लैग्रांजियन खोज (interpretable Lagrangian discovery) ढांचा प्रस्तुत करता है, जिसमें लैग्रांजियन विवरणों को डेटा से प्रत्यक्ष रूप से स्पष्ट और संरचित रूप में पहचाना जाता है। प्राप्त मॉडलों को शास्त्रीय मॉडल प्रेडिक्टिव कंट्रोल (MPC) के साथ एकीकृत करके, एक मॉडल-निरपेक्ष MPC ढांचा विकसित किया गया है, जो आंशिक रूप से या पूर्णतः अज्ञात परिवेशों में भविष्यवाणी नियंत्रण को संभव बनाता है। यह दृष्टिकोण निरंतर ऑनलाइन मॉडल अद्यतन की आवश्यकता को समाप्त करता है, जिससे MPC जटिल वास्तविक-विश्व प्रणालियों के लिए न केवल व्यवहारिक बल्कि प्रभावी भी बन जाता है।

यद्यपि प्रस्तावित मॉडल-निरपेक्ष, डेटा-चालित MPC ढांचा बार-बार ऑनलाइन मॉडल अद्यतन की आवश्यकता को समाप्त करता है, फिर भी इसका वास्तविक-समय में अनुप्रयोग खोजी गई भौतिकी को उच्च-निष्ठा सॉल्वरों के माध्यम से बार-बार अनुकरण करने की संगणनात्मक लागत से सीमित रहता है। यह सीमा विशेष रूप से भविष्यवाणी नियंत्रण

में अधिक महत्वपूर्ण हो जाती है, जहाँ प्रत्येक नियंत्रण चरण पर सीमित समय क्षितिज में प्रणाली के विकास का मूल्यांकन करना आवश्यक होता है। इस संगणनात्मक अवरोध को दूर करने के लिए, यह शोधप्रबंध वेवलेट न्यूरल ऑपरेटर (Wavelet Neural Operator, WNO) प्रस्तुत करता है, जो एक ऑपरेटर लर्निंग ढांचा है और फंक्शन स्पेसों के बीच एक प्राचलीकृत मैपिंग सीखता है, जिससे जटिल स्थान-कालिक गतिकी का तीव्र अनुमान संभव हो पाता है। वेवलेट्स के समय-आवृत्ति स्थानीयकरण गुणों का उपयोग करके, WNO विकसित होती हुई स्थानिक विशेषताओं को प्रभावी रूप से अभिव्यक्त करता है। इसके अतिरिक्त, वेवलेट-डोमेन कन्वोल्यूशन को ग्रीन के प्रमेय से प्रेरित समाकल निरूपण के साथ संयोजित करके, WNO एक निहित ग्रीन कर्नेल सीखता है, जो नॉनलाइनियर सक्रियण फलनों के साथ मिलकर अंतर्निहित नॉनलाइनियर ऑपरेटरों का एक कुशल और सटीक सन्निकटन प्रदान करता है।

WNO पर आधारित होकर, यह शोधप्रबंध ऑपरेटर लर्निंग में डेटा-दक्षता और सामान्यीकरण से संबंधित चुनौतियों को संबोधित करने हेतु मल्टी-फिडेलिटी वेवलेट न्यूरल ऑपरेटर (Multi-Fidelity Wavelet Neural Operator, MF-WNO) तथा न्यूरल कॉम्बिनेटोरियल वेवलेट न्यूरल ऑपरेटर (Neural Combinatorial Wavelet Neural Operator, NCWNO) प्रस्तुत करता है। MF-WNO महंगे उच्च-निष्ठा डेटा तथा कम लागत वाले निम्न-निष्ठा डेटा (जैसे घटित-आदेश मॉडलों या मोटे विविक्तीकरणों से प्राप्त डेटा) के बीच सहसंबंध का उपयोग करके सटीक उच्च-निष्ठा समाधान ऑपरेटरों का अधिगम करता है। समानांतर रूप से, NCWNO अनेक समाधान मैपिंग्स पर एक संयोजकीय ऑपरेटर सीखकर डेटा की आवश्यकता को कम करता है तथा न्यूनतम अतिरिक्त डेटा के साथ नए भौतिक तंत्रों के प्रति तीव्र अनुकूलन करने में सक्षम होता है। इस प्रकार, यह एक आधारभूत (foundation-like) न्यूरल ऑपरेटर के रूप में कार्य करता है, जो आंशिक अवकल समीकरणों के विभिन्न परिवारों में सामान्यीकरण करने की क्षमता रखता है। संगणनात्मक रूप से महंगे सॉल्वरों के बार-बार मूल्यांकन को WNO-आधारित सरीगेट मॉडलों से प्रतिस्थापित करके, प्रस्तावित ढांचा त्वरित योजना निर्माण, कुशल अनुकूलन तथा प्रारंभ से पुनः प्रशिक्षण के बिना निरंतर अधिगम को संभव बनाता है, और इस प्रकार सरीगेट-आधारित भविष्यवाणी नियंत्रण के लिए एक मापनीय तथा संगणनात्मक रूप से दक्ष प्रतिमान स्थापित करता है।

यद्यपि न्यूरल-ऑपरेटर-संवर्धित मॉडल-निरपेक्ष MPC, शास्त्रीय MPC की तुलना में दक्षता में सुधार करता है, फिर भी इसका वास्तविक-समय अनुप्रयोग ऑनलाइन अनुकूलन की लागत से सीमित रहता है। डीप रिइनफोर्समेंट लर्निंग (DRL) प्राचलीकृत नीतियों के माध्यम से एक वैकल्पिक दृष्टिकोण प्रदान करता है, जिन्हें अनुमान (inference) के दौरान सीधे निष्पादित किया जा सकता है; तथापि, पूर्णतः डेटा-चालित DRL अत्यधिक नमूना-गहन होता है, जबकि मॉडल-आधारित रूपांतर ऑनलाइन मॉडल अधिगम से उत्पन्न अशुद्धियों से प्रभावित रहते हैं। इन सीमाओं को संबोधित करने हेतु, यह शोधप्रबंध मॉडल-निरपेक्ष प्रेडिक्टिव टेम्पोरल डिफरेंस (Model-Agnostic Predictive Temporal Difference, MAP-TD) ढांचा प्रस्तुत करता है, जो विरल बेयसियन अधिगम (sparse Bayesian learning) का उपयोग करके ऑफलाइन खोजे गए, पूर्व-प्रशिक्षित, व्याख्येय तथा संक्षिप्त गतिकी मॉडल का लाभ उठाता है। इससे दीर्घकालिक भविष्यवाणी तथा टेम्पोरल-डिफरेंस अनुकूलन के माध्यम से दक्ष नीति-अधिगम संभव हो पाता है, जिसे एक टर्मिनल वैल्यू फंक्शन द्वारा और सुदृढ़ किया गया है। इसके अतिरिक्त, प्रशिक्षण को तीव्र करने के लिए उच्च-निष्ठा सॉल्वरों तथा WNO-आधारित सरीगेट को संयोजित करने वाली एक संकर सिमुलेशन रणनीति अपनाई गई है, जिसके परिणामस्वरूप अंतःक्रिया-रहित तथा नमूना-दक्ष नियंत्रण संभव हो पाता है।

अंततः, प्रस्तावित MAP-TD ढांचे की व्यवहार्यता को दो बड़े पैमाने की बेंचमार्क समस्याओं पर प्रदर्शित किया गया है: एक लचीली गगनचुंबी इमारत में पवन-प्रेरित कंपन न्यूनीकरण तथा सिलेंडर के चारों ओर प्रवाह में प्रवाह-प्रेरित कंपन नियंत्रण। दोनों ही मामलों में, MAP-TD ऐसी नियंत्रण नीतियाँ सीखता है जिन्हें बिना पुनः प्रशिक्षण के सीधे तैनात किया जा सकता है, जिससे वास्तविक-समय नियंत्रण संभव होता है। स्थायी भविष्यवाणी क्षमता वाले एक संक्षिप्त गतिकी मॉडल का उपयोग करके, यह ढांचा ऑनलाइन मॉडल अद्यतन की आवश्यकता को समाप्त करता है तथा प्रशिक्षण के दौरान पर्यावरण के साथ प्रत्यक्ष अंतःक्रिया से बचता है, जिसके परिणामस्वरूप नमूना-दक्षता में उल्लेखनीय सुधार प्राप्त होता है। मात्रात्मक रूप से, MAP-TD शास्त्रीय MPC की तुलना में लगभग 3000 गुना अधिक तीव्र अनुमान (inference) प्राप्त करता है, जबकि समान या बेहतर नियंत्रण प्रदर्शन बनाए रखता है। यह परिणाम अज्ञात गतिकी वाले उच्च-आयामी तंत्रों में कुशल नियंत्रण के लिए इसकी प्रभावशीलता को रेखांकित करता है।

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Abbreviations

CNN	Convolutional Neural Network
DWT	Discrete Wavelet Transform
DRL	Deep Reinforcement Learning
DeepONet	Deep Operator Network
DOF	Degree-of-Freedom
FFNN	Fully connected Neural Network
FNO	Fourier Neural Operator
MAP-TD	Model-Agnostic Predictive Temporal Difference
MCMC	Markov Chain Monte Carlo
MCS	Monte Carlo Simulation
MDOF	Multi Degree-of-Freedom
MDP	Markov Decision Proces
MPC	Model Predictive Control
MPPI	Model Predictive Path Integral
MF-WNO	Multi-Fidelity Wavelet Neural Operator
NO	Neural Operator
NCWNO	Neural Combinatorial Wavelet Neural Operator
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PDF	Probability Density Function
PIP	Probability Inclusion Probability
RL	Reinforcement Learning
SBL	Sparse Bayesian Learning
TD	Temporal Difference
WIB	Wavelet Integral Block
WNO	Wavelet Neural Operator