

A THESIS
ON
NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS
WITH PERIODIC SOLUTION

By

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submitted to the Indian Institute of Technology, Delhi
for the award of the degree, Doctor of Philosophy
in Mathematics

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CERTIFICATE

This is to certify that the thesis entitled, "Numerical Methods for Differential Equations With Periodic Solution", which is being submitted by Mr. U. Anantha Krishnaiah for the award of the degree, Doctor of Philosophy (Mathematics), to the Indian Institute of Technology, Delhi, is a bonafide record of research work. He has worked for the last three years and two months under my guidance and supervision.

The thesis has reached the standard, fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

MK Jain 16/11/79
(M.K. Jain)

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