MANIFESTING BIPOLARITY IN MULTI-OBJECTIVE FLEXIBLE LINEAR PROGRAMMING

by

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Certificate

This is to certify that the thesis entitled **"Manifesting Bipolarity in Multi-objective Flexible Linear Programming"** submitted by **Ms. Dipti Dubey** to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bona fide research work carried out by her under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Abstract

Linear programming (LP) traditionally works with hard constraints, but in many circumstances flexible version of constraints may be beneficial when specifying strict satisfaction of constraints leads to inconsistencies resulting in the vacuousness of the feasible set. A flexible linear program (FLP) may be of great help in such situations. Moreover, if there are more than one objective functions to be optimized simultaneously in an FLP problem then it is termed as a multi-objective FLP (MOFLP) problem. The MOFLP problems are often investigated under the name fuzzy multi-objective linear programming because fuzzy sets are used as principal tool to mathematically represent flexible constraints.

The traditional framework for MOFLP problems is inspired by the max-min approach proposed by Zimmermann. This approach consists of mainly two steps. In the first step objective functions are transformed into the flexible constraints using aspiration levels. In the second step the fuzzy sets representing the flexible constraints corresponding to the objective functions and the original flexible constraints of MOFLP are aggregated together to define the decision set.

In this thesis, we propose an alternative approach to model MOFLP from a perspective of heterogeneous bipolar view in preference modeling. The flexible constraints in a MOFLP are viewed as negative preferences for describing what is somewhat tolerable while the flexible constraints corresponding to the objective functions of the problem are viewed as positive preferences for depicting satisfaction to what is desirable. This viewpoint enables us to handle fuzzy sets representing constraints and objective functions separately and combine them in distinct ways. After aggregating these fuzzy sets separately, the coherence (or consistency) condition is used to define the ultimate decision set and a new 'bipolar approach' is proposed to solve MOFLP problems.

This bipolar approach is further extended to multi-objective flexible linear programming with interval uncertainty (MOFLPIU). A flexible constraint with interval uncertainty extends the flexible constraint by allowing preferences to be expressed in the form of intervals. In order to accomplish this aim, in the first place, we study flexible linear programming with interval uncertainty (FLPIU). Using mathematical equivalence of interval-valued fuzzy set and Attanasov's intuitionistic fuzzy set (I-fuzzy set), interval uncertainty in a flexible constraint is expressed mathematically using an I-fuzzy set. According to construction of non-membership function, three different viewpoints viz., optimistic, pessimistic, and mixed, are proposed to interpret an interval uncertainty. The solution methods are proposed to solve FLPIU by associating fuzzy sets with each I-fuzzy set representing flexible constraints with interval uncertainty. Afterwards we study MOFLPIU and integrate the heterogenous bipolarity in it. We propose an optimistic-pessimistic approach to solve MOFLPIU using an interval-valued fuzzy set representation and the Hurwicz optimism-pessimism criterion.

Finally in this thesis, we define a solution concept of Pareto-optimality for MOFLP problems. An approach is proposed to single out such a solution for MOFLP with highest possible degree of feasibility in the bipolar framework. The bipolar approach to solve MOFLP is restudied with consideration to this solution concept and an approach is proposed to extract Pareto-optimal solution from the set of optimal solutions of its deterministic equivalent optimization problem. The solution concept of Pareto-optimality is also extended to MOFLPIU problems in such a way that a Pareto-optimal solution of MOFLPIU can be obtained using the approach proposed to extract the Pareto-optimal solution of MOFLP.

Contents