

Efficient Parallel Output-Size Sensitive Algorithms

by

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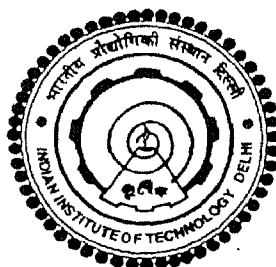
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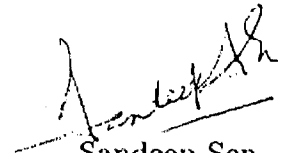


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Certificate

This is to certify that the thesis titled "Efficient Parallel Output-Size Sensitive Algorithms" being submitted by Neelima Gupta to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy, is a record of bonafide research carried out by her under my supervision. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any other degree or diploma.


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Abstract

We present efficient PRAM algorithms whose performance is related to the size of the output of the given instance (of a problem). For a number of geometric problems, the output-size is often smaller than the worst-case behaviour in terms of the input-size. With careful design, output-sensitive algorithms exhibit superior performance for these problems than the worst-case optimal algorithms. We address the following problems: Convex hulls in two and three dimensions, Vector maxima in two and three dimensions and Hidden surface elimination problem for a special class of surfaces called terrains.

Most of our algorithms use CRCW PRAM model and improve the previously best-known results for these problems in terms of running time, and/or work-bound. Some of our algorithms exploit randomized techniques to provide faster solutions. For convex hulls in two dimensions our algorithms are faster for small output sizes than the previously known output sensitive algorithms keeping the work optimal. In three dimensions our algorithm is faster for all output sizes. We obtain similar results for vector-maxima in two and three dimensions. To the best of our knowledge, no previous parallel output-sensitive algorithm was known for 3-D vector maxima. We also present sublogarithmic-time algorithms that use superlinear number of processors for vector maxima in three dimensions. For the hidden surface elimination problem for terrains, our algorithm is simpler than the previous work and is more complete in its description of the details. Moreover, our algorithm works for a weaker model.

Further, we present some optimal parallel distribution sensitive algorithms for planar hulls, vector maxima and multiset sorting problems which provide superior solutions than the previous output-sensitive algorithms.

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