INITIAL EXCITATION BASED ADAPTIVE OBSERVER FOR UNCERTAIN SYSTEMS

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INITIAL EXCITATION BASED ADAPTIVE OBSERVER FOR UNCERTAIN SYSTEMS

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Dedicated to friends and family...

Certificate

This is to certify that the thesis entitled **"Initial Excitation Based Adaptive Observer for Uncertain Systems"**, submitted by **Atul Katiyar** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy**, is a record of the original, bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

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Atul Katiyar

Abstract

Adaptive observer design deals with online estimation of states using input-output information of a dynamical system in the presence of parametric uncertainty in the dynamics. It works with the principle of simultaneous estimation of states and the uncertain parameters using suitable online estimation algorithms to ensure stability of the estimation error dynamics. Conventional adaptive observers rely on the richness of input-output signals to satisfy the persistence of excitation (PE) condition for parameter convergence. The PE condition is restrictive since it demands sufficient energy of the signal for the entire time span and the condition depends on the future behavior of the signal, which poses difficulty in online verification.

Existing adaptive observer algorithms rely on the PE condition for the convergence of unknown parameters to zero. To enforce the PE condition, it is required that the external auxiliary signal should sustain significant richness for the entire time of operation. Hence, PE is stringent in the sense that it is not always realizable, and it is often impractical to monitor online whether a signal will remain PE as the condition relies on the future behavior of the signal. Therefore, relaxing the PE condition is a long-standing area of research in adaptive control and estimation. In addition, the persistent energy demands of PE may cause damage to many real-world systems, e.g., aircraft systems [1] etc.

For adaptive control problems, the authors in [2, 3, 4, 5, 6, 7] proposed a two-layer filter based new adaptive control architectures, which guarantee parameter convergence under the milder condition of initial excitation (IE). The IE condition requires that the regressor has sufficient energy in the initial time window of finite length, which is less restrictive than the PE condition, where the regressor is required to have sufficient energy for the entire time span. Further, the IE condition can be verified online and may be ensured by injecting a transient dither-like signal.

This dissertation presents an IE-based adaptive observer architecture that ensures parameter convergence in the absence of PE. A switched estimator based on a double-layered filter architecture is used which by virtue of switching captures the necessary information (about unknown parameters) from the transient cycles and uses it for future references in the absence of PE.

सार

अनुकूली पर्यवेक्षक डिजाइन का संबंध गतिशील प्रणाली की इनपुट-आउटपुट जानकारी का उपयोग करके पैरामीट्रिक अनिश्चितता की उपस्थिति में अवस्थाओं के ऑनलाइन अनुमान से है। यह उपयुक्त ऑनलाइन अनुमान कलन विधि का उपयोग करके अवस्थाओं और अनिश्चित मापदंडों के एक साथ अनुमान के सिद्धांत के साथ काम करता है ताकि अनुमान त्रुटि गतिकी की स्थिरता सुनिश्चित हो सके। पारंपरिक अनुकूली पर्यवेक्षक मापदंड अभिसरण के लिए उत्तेजना की दृढ़ता (PE) की स्थिति को संतुष्ट करने के लिए इनपुट-आउटपुट संकेतों की समृद्धि पर निर्भर करते हैं। PE की स्थिति प्रतिबंधात्मक है क्योंकि यह पूरे समय अवधि के लिए संकेत की पर्याप्त ऊर्जा की मांग करती है और यह स्थिति संकेत के भविष्य के व्यवहार पर निर्भर करती है, जो ऑनलाइन सत्यापन में कठिनाई पैदा करती है।

मौजूदा अनुकूली पर्यवेक्षक एल्गोरिदम अज्ञात मापदंडों के शून्य में अभिसरण के लिए PE की स्थिति पर निर्भर करते हैं। PE की स्थिति को लागू करने के लिए, यह आवश्यक है कि बाहरी सहायक संकेत पूरे समय संचालन के लिए पर्याप्त समृद्धि बनाए रखे। इसलिए, PE कठोर है क्योंकि यह हमेशा वास्तविक नहीं हो सकता है, और यह अक्सर ऑनलाइन निगरानी के लिए अव्यावहारिक होता है कि कोई संकेत PE रहेगा या नहीं क्योंकि यह स्थिति संकेत के भविष्य के व्यवहार पर निर्भर करती है। इसलिए, अनुकूली नियंत्रण और अनुमान में PE की स्थिति को शिथिल करना एक दीर्घकालिक अनुसंधान क्षेत्र है। इसके अलावा, PE की स्थायी ऊर्जा की मांगें कई वास्तविक-विश्व प्रणालियों को नुकसान पहुंचा सकती हैं, जैसे विमान प्रणालियाँ।

अनुकूली नियंत्रण समस्याओं के लिए, लेखकों ने [2, 3, 4, 5, 6, 7] में एक दो-स्तरीय फिल्टर आधारित नया अनुकूली नियंत्रण आर्किटेक्चर प्रस्तावित किया है, जो प्रारंभिक उत्तेजना (IE) की हल्की स्थिति के तहत मापदंड अभिसरण की गारंटी देता है। IE की स्थिति की आवश्यकता होती है कि प्रारंभिक समय विंडो की सीमित लंबाई में पुनरावृत्ति के पास पर्याप्त ऊर्जा हो, जो PE की स्थिति की तुलना में कम प्रतिबंधात्मक है, जहां पुनरावृत्ति के पास पूरे समय अवधि के लिए पर्याप्त ऊर्जा होनी चाहिए। इसके अलावा, IE की स्थिति को ऑनलाइन सत्यापित किया जा सकता है और इसे एक अस्थायी डिथर-जैसे संकेत को इंजेक्ट करके सुनिश्चित किया जा सकता है।

यह शोध प्रबंध एक IE-आधारित अनुकूली पर्यवेक्षक आर्किटेक्चर प्रस्तुत करता है जो PE की अनुपस्थिति में मापदंड अभिसरण सुनिश्चित करता है। एक दो-स्तरीय फिल्टर आर्किटेक्चर पर आधारित एक स्विच किए गए अनुमानक का उपयोग किया जाता है जो स्विचिंग के माध्यम से आवश्यक जानकारी (अज्ञात मापदंडों के बारे में) अस्थायी चक्रों से प्राप्त करता है और PE की अनुपस्थिति में इसे भविष्य के संदर्भ के लिए उपयोग करता है।

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Abbreviations

PE	**P**ersistence **E**xcitation
F-PE	**F**inite **P**ersistence **E**xcitation
IE	**I**nitial **E**xcitation
S-IE	**S**trong **I**nitial **E**xcitation
LTI	**L**inear **T**ime **I**nvariant
SISO	**S**ingle **I**nput **S**ingle **O**utput
MIMO	**M**ultiple **I**nput **M**ultiple **O**utput
UUB	**U**niformly **U**ltimately **B**ounded
GUUB	**G**lobally **U**niformly **U**ltimately **B**ounded
UGES	**U**niformaly **G**lobal **E**xponential **S**table

Symbols

$\mathbb{R}$	real space
$\mathbb{R}^{n\times n}$	real space of matrices of dimension $n\times n$
$\subset$	subset
$\in$	belongs to
$\forall$	for all
$\exists$	there exists
$\emptyset$	null space
s	switch s
λ_M	maximum eigen value of ${\Gamma_\theta}^{-1}$
λ_m	minimum eigen value of ${\Gamma_\theta}^{-1}$
$\lambda_{min}(Q)$	minimum eigen value of matrix Q
Ω_{PE}	the space of PE signals
Ω_{IE}	the space of IE signals
$\Omega_{\mathcal{L}_2}$	the space of square-integrable signals
$\Omega_{\mathcal{L}_\infty}$	the space of bounded signals
$[\cdot]$	box function