

**OPTIMAL ARRAY PROCESSING FOR SPHERICAL  
SECTOR MICROPHONE ARRAY**

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**DEPARTMENT of ELECTRICAL ENGINEERING  
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**OPTIMAL ARRAY PROCESSING FOR SPHERICAL  
SECTOR MICROPHONE ARRAY**

by

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DEPARTMENT of ELECTRICAL ENGINEERING

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to the



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### CERTIFICATE

This is to certify that the thesis entitled “**Optimal Array Processing for Spherical Sector Microphone Array**” being submitted by **Ms. Deepika Kumari** to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is the record of bonafide research work carried out by her under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma

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**Dedicated  
To  
My Mother,  
Mrs. Radhika Sharma**

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# Abstract

Spherical microphone array (SMA) signal processing has received much attention in the recent years for applications like higher-order sound field capture, analysis of room acoustics, direction of arrival (DOA) estimation, beamforming, and source separation. The widespread application of SMAs is due to the ease of array processing in the spherical harmonics (SH) domain without spatial ambiguity. Utilization of SH allows separation of variables for functions of DOA, sensor position, and wavenumber, thus facilitating frequency smoothing and dimensionality reduction. However, the construction of an SMA over a rigid sphere is a challenging task. Additionally, utilization of an entire sphere comes at the expense of a greater number of microphones and signals to process. It is also uneconomical to use an SMA when sources are present in restricted regions of the environment. Attempts have been made to use hemispherical microphone arrays on the basis of the acoustic image principle, enabling application of SH but with greater computational complexity.

As a part of the thesis, utilization of spherical sector microphone array is proposed. An orthonormal spherical sector harmonics ( $S^2H$ ) basis function is developed for accurate representation of pressure over the sector. The orthonormality of the  $S^2H$  function is established using orthogonality of shifted associated Legendre polynomials and a scaled exponential function. The discontinuity over the boundary is handled by solving the Helmholtz equation over the sector. An expression for the sectoral mode strength for an open and rigid spherical sector is derived based on exact solution to the wave equation. An addition theorem for  $S^2H$  basis functions is established to account for an incident field angle in the plane-wave solution to the wave equation. The norm of the  $S^2H$  basis function is computed using  $S^2H$  addition theorem.

Subsequently, source localization, beamforming, and source separation problems are formulated in spherical sector harmonics domain. The new  $S^2H$  basis functions are utilized for far-field and mixed-field data model development. An ideal direction-invariant beampattern and maximum directivity white noise gain (MD-WNG) beamformer are proposed for the spherical sector microphone array. Simulation and real experiment results are presented for narrowband and wideband signals.

# सारांश

गोलाकार माइक्रोफोन सरणी (एसएमए) सिग्नल प्रोसेसिंग ने हाल के वर्षों में उच्च-क्रम वाले ध्वनि क्षेत्र पर कब्जा, कमरे के ध्वनिकी के विश्लेषण, आगमन की दिशा (डीओए) अनुमान, बीमफॉर्मिंग और स्रोत पृथक्करण जैसे अनुप्रयोगों के लिए बहुत ध्यान दिया है। एसएमए का व्यापक अनुप्रयोग स्थानिक अस्पष्टता के बिना गोलाकार हार्मोनिक्स (एसएच) डोमेन में सरणी प्रसंस्करण की आसानी के कारण है। एसएच का उपयोग डीओए, सेंसर स्थिति और वेवनंबर के कार्यों के लिए चर को अलग करने की अनुमति देता है, इस प्रकार आवृत्ति चौरसाई और आयामी कमी की सुविधा प्रदान करता है।

हालांकि, एक कठोर क्षेत्र में एक एसएमए का निर्माण एक चुनौतीपूर्ण कार्य है। इसके अतिरिक्त, एक संपूर्ण क्षेत्र का उपयोग प्रक्रिया के लिए अधिक संख्या में माइक्रोफोन और सिग्नल की कीमत पर आता है। पर्यावरण के प्रतिबंधित क्षेत्रों में स्रोत मौजूद होने पर एसएमए का उपयोग करना भी आर्थिक नहीं है। ध्वनिक छवि सिद्धांत के आधार पर अर्धगोलाकार माइक्रोफोन सरणियों का उपयोग करने का प्रयास किया गया है, एसएच के अनुप्रयोग को सक्षम करता है लेकिन अधिक कम्प्यूटेशनल जटिलता के साथ।

थीसिस के एक भाग के रूप में, गोलाकार सेक्टर माइक्रोफोन सरणी का उपयोग प्रस्तावित है। क्षेत्र पर दबाव के सटीक प्रतिनिधित्व के लिए एक ऑर्थोनॉर्मल गोलाकार सेक्टर हार्मोनिक्स आधार फंक्शन विकसित किया गया है। गोलाकार सेक्टर हार्मोनिक्स फंक्शन की ऑर्थोनॉर्मलिटी को शिफ्ट किए गए संबद्ध लेजेन्ड्रे बहुपदों की ऑर्थोगोनैलिटी और एक स्केल किए गए एक्सपोनेंशियल फंक्शन का उपयोग करके स्थापित किया गया है। क्षेत्र पर हेलमहोल्ट्ज़ समीकरण को हल करके सीमा पर असंतुलन को नियंत्रित किया जाता है। एक खुले और कठोर गोलाकार क्षेत्र के लिए क्षेत्रीय मोड ताकत के लिए अभिव्यक्ति तरंग समीकरण के सटीक समाधान के आधार पर प्राप्त की जाती है। गोलाकार सेक्टर हार्मोनिक्स आधार कार्यों के लिए एक अतिरिक्त प्रमेय को तरंग समीकरण के समतल-तरंग समाधान में एक घटना क्षेत्र कोण के हिसाब से स्थापित किया जाता है। गोलाकार सेक्टर हार्मोनिक्स आधार फंक्शन के मानदंड की गणना गोलाकार सेक्टर हार्मोनिक्स जोड़ प्रमेय का उपयोग करके की जाती है।

इसके बाद, गोलाकार क्षेत्र हार्मोनिक्स डोमेन में स्रोत स्थानीयकरण, बीमफॉर्मिंग और स्रोत पृथक्करण समस्याएं तैयार की जाती हैं। नए गोलाकार सेक्टर हार्मोनिक्स आधार कार्यों का उपयोग दूर-क्षेत्र और मिश्रित-क्षेत्र डेटा मॉडल विकास के लिए किया जाता है। गोलाकार क्षेत्र माइक्रोफोन सरणी के लिए एक आदर्श दिशा-अपरिवर्तनीय बीम पैटर्न और अधिकतम प्रत्यक्षता सफेद शोर लाभ (एमडी-डब्ल्यूएनजी) बीमफॉर्मर प्रस्तावित हैं। नैरोबैंड और वाइडबैंड सिग्नल के लिए सिमुलेशन और वास्तविक प्रयोग के परिणाम प्रस्तुत किए जाते हैं।

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# List of Symbols

$(.)^*$	Complex conjugate of scalar $(.)$
$(.)^T$	Transpose of vector or matrix $(.)$
$(.)^H$	Conjugate transpose of matrix or vector $(.)$
<b>A</b>	Steering matrix in spatial domain
<b>B</b>	Mode strength matrix
$\tilde{b}_n(kr)$	$n$ th order mode strength for spherical sector
*	Convolution
$c$	Speed of sound
$\tilde{e}^{jm\phi}$	Scaled exponential function
$h_n^{(2)}(kr)$	Spherical Hankel function of second kind
$i$	Microphone index
$I$	Number of Microphones
$j$	Unit imaginary number
$j_n(kr)$	Spherical Bessel function of first kind
$k$	Wavenumber
<b>k</b>	Wavevector
$l$	Source index
$L$	Number of sources
$m$	Degree (of spherical harmonics or spherical sector harmonics)
$n$	Order (of spherical harmonics or spherical sector harmonics or mode strength)
$N$	Order of spherical microphone array or spherical sector microphone array
$N_s$	Number of snapshots
$p(n_t)$	Pressure in discrete time domain
$P(\nu, \tau)$	Short-time Fourier transform of pressure $p(n_t)$
$P_{nm}(\nu, \tau)$	Spherical sector harmonics coefficients
$\mathcal{P}_n^m(x)$	Associated Legendre polynomials
$\tilde{\mathcal{P}}_n^m(x)$	Shifted associated Legendre polynomials

$\mathbf{Q}_{nm}$	Noise subspace
$\mathbf{r}$	Position vector
$\mathbf{R}_{\mathbf{D}_{nm}}$	Array covariance matrix
$r$	Radius of spherical array or spherical sector microphone array
$T_n^m$	Spherical sector harmonics of order $n$ and degree $m$
$W_i$	Beamforming weight at $i$ th microphone
$\mathbf{w}(k)$	Beamforming coefficient/weight vector in space domain
$\mathbf{w}_{nm}(k)$	Beamforming coefficient/weight vector in $S^2H$ domain
$Y_n^m$	Spherical harmonics of order $n$ and degree $m$
$\mathbf{Z}_{nm}(k)$	Beamformed output in $S^2H$ domain
$\nabla$	Gradient
$\tau$	Time frame index
$\nu$	Frequency bin index
$\Psi$	Angular location of a source
$\Phi$	Angular location of a microphone
$\theta$	Elevation angle
$\phi$	Azimuth angle
$\lambda$	Wavelength
$\zeta$	Confidence interval
$\gamma$	Angle between the plane wave and microphone

# List of Abbreviations

2D	Two dimensional
3D	Three dimensional
AIP	Acoustic image principle
ALP	Associated Legendre polynomial
CRMSE	Cumulative root mean square error
DOA	Direction of Arrival
DF	Directivity factor
FDM	Finite difference method
HMA	Hemispherical microphone array
HSH	Hemispherical harmonics
MUSIC	MULTiple SIGNAL Classification
MVDR	Minimum Variance Distortionless Response
MF	Mixed- field
MD	Maximum directivity
MD–WNG	Maximum directivity and white noise gain
NB	Narrowband
POR	Probability of resolution
PDE	Partial differential equation
RMSE	Root mean square error
SH	Spherical harmonics
SD	Standard deviation
SHD	Spherical harmonics decomposition
S <sup>2</sup> HD	Spherical sector harmonics decomposition
S <sup>2</sup> H	spherical sector harmonics
SIR	Signal to Interference Ratio
SMA	Spherical Microphone Array
S <sup>2</sup> MA	Spherical sector microphone array

SNR	Signal to Noise ratio
STFT	Short time Fourier transform
WB	Wideband
WNG	White noise gain