

A STUDY OF TERRAIN MODELLING AND FINITE ELEMENT BASED WATERSHED DYNAMICS

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सकल लोकस्य सेवायै समर्पणये

सर्वेहत्र सुखिनः सन्तु

सर्वे सन्तु निरामयाः

सर्वे भद्राणि पश्यन्तु

मा कश्चित् दुःखमाप्नुयात् ।।

CERTIFICATE

This is to certify that the thesis entitled , ' **A STUDY OF TERRAIN MODELLING AND FINITE ELEMENT BASED WATERSHED DYNAMICS** ' by Shri Dhruvajyoti Sen to The Indian Institute of Technology , Delhi , India , for the award of the degree of **DOCTOR OF PHILOSOPHY** , is a record of bonafide research work carried out by him under my supervision and guidance . The thesis work , in my opinion , has reached the standard , fulfilling the requirements for **DOCTOR OF PHILOSOPHY** degree . The research report and the results presented in this thesis have not been submitted , in part or in full , to any other University or Institute , for the award of any degree or diploma .

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ABSTRACT

The mathematical simulation models describe , in various degrees of simplicity or accuracy , the transformation of the input (rainfall) through the watershed to produce the output (runoff) at the point of interest . It is now possible to solve numerically the governing differential equations with the fantastic development in the field of computers , data collection , and communication . The new methods in this direction are explored in the present study .

Terrain modelling will be a pre requisite to hydrologic simulation of the rainfall runoff process . New algorithms are developed in the present study to extract the watershed features , like overland flow cascades , channel network , confluence points , ridges , etc. for a given digital elevation data of scattered points using Triangular Irregular Network (TIN) .

The main temporal input to the watershed is taken as the rainfall data at the different rain gauge stations . A methodology based on the finite element concept is developed to obtain the time invariant weighting coefficients for estimation of the rainfall on the cascade planes .

The governing de Saint Venant equations of flow are solved by finite element method . Kinematic wave approximation is assumed for the overland flow calculations , while the full equations are solved for the channel flows . Linear shape functions are used to approximate the spatial distribution of the variables , while the temporal distribution is approximated by the finite difference technique . The resulting non-linear equations are solved by the Newton Raphson method for every time step . The outflows from all the overland cascade planes are calculated at each time step , and are used as lateral inflows to the channel flows . For the overland flow calculations , an element by element algorithm is developed to solve the non-linear equations using the frontal concept .

In the channel flow computations , the junction conditions of conservation of flow and equivalence of energy introduces the non-zero terms outside the bandwidth . A new algorithm is developed to store and solve the assembled matrix for a given network . It is efficient and can easily be programmed to work on a parallel processor .

The results for the model catchments clearly demonstrate the general applicability of the formulations and concepts presented in this study

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BIO-DATA