

STUDIES OF THE STATE AND PARAMETER ESTIMATION  
IN DISTRIBUTED SYSTEMS

by

K. SUDHAKARA REDDY

Thesis submitted to the Indian Institute of Technology, Delhi  
for the award of

DOCTOR OF PHILOSOPHY

Department of Electrical Engineering  
Indian Institute of Technology, Delhi

New Delhi-110029

April 1976

CERTIFICATE

This is to certify that the thesis, "Studies of the State and Parameter Estimation in Distributed Systems", which is being submitted by K. Sudhakara Reddy for the award of degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my supervision.

This dissertation has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

*V. S. Rajamani*

( V.S. RAJAMANI )  
Department of Electrical Engineering  
Indian Institute of Technology  
New Delhi-110029.

## ACKNOWLEDGEMENTS

I am grateful to Professor V.S. Rajamani for his valuable guidance at all stages of this work. His keen interest in this field has been a constant source of encouragement. I owe a debt of gratitude to him for spending considerable amount of time in reading the thesis manuscript and making numerous suggestions for its improvement.

My thanks are due to my colleagues Dr. J. Mishra and Dr. K.V. Lu for the invaluable benefits that I derived from the discussions I had with them. I would also like to acknowledge the valuable assistance I received from M|s A.N.Jha, T. Jeevanandam, G. Singh, R.M. Prasad and S. Varshney at various stages of this work . My thanks are also to Miss Neelam Dhody, who did a fine job of typing the manuscript.

The financial support received from Council of Scientific and Industrial Research is gratefully acknowledged.

New Delhi,  
April 20, 1976.

K. SUDHAKARA REDDY

ABSTRACT

This thesis addresses to the problem of state and parameter estimation in distributed parameter systems with noisy measurement data. Eventhough the estimation in distributed systems received a great deal of interest in recent years, there are still many unresolved problems that need to receive attention. This study applies some new techniques to resolve the estimation problems in distributed parameter systems. The first of these consists in using the Innovations Approach to obtain the Kalman-Bucy type distributed filter equations when the observations are corrupted by coloured noise. This approach provides an appealingly simple derivation and does not require any discretization procedures. In the second chapter the problem of state estimation for distributed parameter systems that have both the transportation and measurement delays is considered. In many practical situations time-delays are an integral part of the system and have to be taken into account. The use of the innovations-approach provides a simpler derivation for systems with time-delays. As a by-product the smoothing solutions for a non-time-delayed system can be obtained from the estimation equations of the above time-delay system. The solution of the estimation equations for the time-delay system often give rise to complicated partial differential equations that are very difficult to solve. The present study also presents

the consideration of manifold possibilities arising in distributed systems state-estimation problems. Thus the Kalman-Bucy type filters are established for processing measurement-data obtained from spatially-continuous, spatially-independent and spatially-discrete points.

Till now, the studies in the estimation theory of distributed systems were all based on the distributed type matrix-Riccati-equation which requires the solution of  $\sqrt{n(n+1)/2}$  (n being the dimension of the state space) partial differential equations. The Riccati-equation-based estimation results received a great deal of interest, because of their applicability to both time-varying and time-invariant systems. For time-invariant distributed systems an alternate algorithm that avoids the solution of Riccati equation is presented. This algorithm has many computational advantages depending upon the dimension of the measurement equation and the rank of the initial covariance matrix. Two special cases are considered viz. (i) low initial uncertainty and (ii) high initial uncertainty. These special cases are both interesting and has many computational advantages.

The problem of state estimation in interacting systems with lumped and distributed parameters is considered next. This problem is very important in many chemical processes. Since many of these systems are nonlinear in character, this thesis considers the nonlinear interacting systems. (However the case of linear interacting systems which is a particular case is

presented in the Appendix). Due to nonavailability of sufficient results in the area of stochastic calculus for distributed systems, a direct extension of innovations approach to nonlinear distributed systems does not seem to be possible. Hence we pose this nonlinear estimation problem in an essentially deterministic framework and use the optimal control theory. The optimal control problem is recast in the hierarchical framework, so that it is easier to deal with subsystems of lower dimension than one large system of higher dimension. The two-point boundary value problem that arises in the solution of the optimal control problem is solved using the Invariant Imbedding method. An example of recycle chemical reactor is presented.

Lastly, the problem of parameter estimation of distributed parameter system is considered. In the present study, the aim is not to go for any new method for parameter estimation, but to develop a new method of modelling the distributed system by a lumped system so that the known techniques can be used for parameter estimation. The finite element method, which is popular with the Structural Engineers is used here to model the distributed system by a lumped system. After making the finite element approximation, the extended Kalman filter is used to estimate the parameters. Two numerical examples are presented with a view to bring out the salient features of the finite element method.

-----

## TABLE OF CONTENTS

	<u>Page</u>
	(ii)
ACKNOWLEDGEMENTS	(iii)
ABSTRACT	(iii)
I	1
INTRODUCTION	1
1.1	1
1.2	7
1.3	10
1.3.1	13
1.3.2	15
1.3.3	17
1.4	18
II	21
OPTIMAL LINEAR FILTERING WITH COLOURED MEASUREMENT NOISE	21
2.1	21
2.2	23
2.3	26
2.3.1	30
2.3.2	34
2.3.3	38
2.3.4	40
2.4	43
2.4.1	45
2.5	47
III	49
OPTIMAL LINEAR FILTERING WITH TIME-DELAYS	49
3.1	49
3.2	50
3.3	53
3.3.1	56
3.3.2	61
3.3.3	63
3.4	65
3.5	65
3.6	67

		<u>Page</u>
IV	A NEW FILTERING ALGORITHM	68
	4.1 Introduction	68
	4.2 Chandrasekhar Type Equations	72
	4.2.1 Low Initial Uncertainty	78
	4.2.2 High Initial Uncertainty	79
	4.2.3 Measurements at Discrete Points in the Spatial Domain	83
	4.2.4 Spatially-Independent Measurements	84
	4.3 Solution of the New Filter Equations	85
	4.4 The Smoothing Solution	92
	4.5 Concluding Remarks	92
V	FILTERING IN INTERACTING SYSTEMS WITH LUMPED AND DISTRIBUTED PARAMETERS	94
	5.1 Introduction	94
	5.2 Problem Formulation	97
	5.3 Derivation of the Filter	99
	5.3.1 Decomposition Into Subsystems	101
	5.4 Sequential Coordination	109
	5.5 Numerical Example	111
	5.6 Concluding Remarks	115
VI	PARAMETER ESTIMATION USING FINITE ELEMENT METHOD	117
	6.1 Introduction	117
	6.2 Problem Formulation	121
	6.3 Finite Element Method	123
	6.4 Extended Kalman Filtering	131
	6.5 Numerical Examples	134
	6.6 Concluding Remarks	139
VII	CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK	140
	7.1 A Brief Summary of Results	140
	7.2 Suggestions for Further Work	142
	APPENDIX	144
	REFERENCES	150