FOURIER PHASE RETRIEVAL:
NEW STRATEGIES AND APPLICATIONS TO COHERENT DIFFRACTION IMAGING

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MAY 2023

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NEW STRATEGIES AND APPLICATIONS TO COHERENT DIFFRACTION IMAGING

by

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submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MAY 2023

Dedicated to

my teachers

Haimanti and Prof. Kedar

and my parents

CERTIFICATE

This is to certify that the thesis entitled, "**Fourier Phase Retrieval: New Strategies and Applications to Coherent Diffraction Imaging**", being submitted by **Mansi Butola**, to the Indian Institute of Technology Delhi, for the award of degree of **Doctor of Philosophy** in the Department of Physics is a record of bonafide research work carried out by her under my supervision and guidance. She has fulfilled the requirements for the submission of the thesis, which to the best of my knowledge has reached the required standard.

The material contained in the thesis has not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Date: May 2023
Place: New Delhi

ACKNOWLEDGEMENTS

I want to take this opportunity to sincerely thank everyone who has helped me and been so kind with their time and energy.

First, I would like to express my profound gratitude to my PhD adviser, **Prof. Kedar Khare**, for his excellent guidance, consistent positivity, and kind encouragement during the last five years. I owe a lot to him for all the energy he put in, the things he taught us, and the meticulous care he took over every aspect of our projects. His sharp and observant approach of conducting the research played very important role in completing this thesis. I count myself fortunate to have had him as a PhD supervisor, which made my five-year PhD journey worthwhile. **Prof. D.S. Mehta**, **Prof. Joby Joseph**, **Prof. Sesan Srirangarajan**, and **Prof. Kedar khare**, all of whom sat on my Student Research Committee throughout my PhD, have my deepest gratitude for their patient guidance and insightful feedback. I would also like to thank **Prof. P. Senthilkumaran** and **Prof. K. Thyagarajan** for providing me with interesting and engaging lectures in Optics. I am thankful to the Indian Institution of Technology Delhi for giving me with a five-year institute assistanceship as well as several opportunities and financial support to attend national and international conferences. I am also thankful to the IIT Delhi hostel staff, who looked after the hostel residents in every possible way.

I would like to express my humble gratitude to the educators in school who helped shape my formative years at Jawahar Navodaya Vidyalaya, Rudraprayag: **Mr. K.N. Upadhyaya**(our school principal), **Dr. Deepa Upadhyaya**, and **Mr. Mahesh Bhardwaj**. My heartfelt gratitude goes to **Prof. Raksha Sharma** for being a wonderful mentor and friend throughout my undergraduate studies at Kirori Mal College, University of Delhi. I am immensely thankful to **Dr. Sucheta Padaliya** and **Ustad Wasifuddin Dagar** for offering excellent Indian classical music lessons that have added rhythm to my life and made these years even more delightful.

I am grateful beyond words to my parents, **Mrs. Meena Butola** and **Mr. Yashwant Butola**, my grandma and aunt, **Mrs. Saruli Rana** and **Ms. Deena Rana**, for all the love and support they poured on me. Thanks to my sister **Pragya** and brother **Akhil**, I have people who always have my back and cheer me on no matter what. I would also want to express my gratitude to my maternal family, **Mrs. Sarojini Rana, Mrs. Deepa Rana, Mr. Dhirendra Rana**, and **Mr. Ajay Rana**, cousins **Yashmi, Anirudh,**

Akshat, and **Ayush** for all the times they have shown me affection and cared for me while I was away from home.

I thank my childhood friends from the school and college friends: **Poonam, Pooja, Shark, Nikita, Jigyasa, Ayush, Monika, Pragati, Anisha, Hemant** and **Dhriti** for their pleasant and joyous companionship all these years. I really thank my labmates **Sunaina, Jasleen Kaur** for our stimulating discussions on many topics of inverse imaging problems. I also consider myself lucky to be in the good company of friends, **Sunaina, Milton**, and **Shilpa**, who were there for me through thick and thin and shared their merits with me.

Ultimately, I express my heartfelt gratitude to the divine, the all-encompassing cosmic force for this wonderful existence, for fulfilling all my necessities, and surrounding me with compassionate and supportive individuals. I am particularly grateful for the presence of spiritual masters- **Haimanti Mukhopadhyay, Jaggi Vasudev, and S.N. Goenka** - in my lifetime, who generously imparted the incredible knowledge of yoga and meditation. These practices have completely transformed my inner being.

New Delhi — Mansi Butola

2 January 2023 — (2016PHZ2024)

ABSTRACT

Retrieving phase of a signal from measurements of its magnitude in some transform domain is an important problem in many disciplines of Science and Engineering. One of the attractive (non-interferometric) phase imaging techniques that is generally handled computationally using iterative methods is by measuring the far-field or Fraunhofer diffraction intensity of an object, which is known as Fourier phase retrieval. The Fourier phase retrieval problem is non-convex and non-linear in nature, which makes it challenging to find a unique solution. Iterative transform algorithms have been developed for almost a half-century and are a popular option among users. However, these algorithms are incapable of providing a high-quality and reliable solution in a single-run starting with a random initial guess. The phase retrieval solution, in particular, is degraded by many stagnation artifacts that may be attributed to inaccuracy in the recovered Fourier phase or the presence of missing pixels and noise in Fourier intensity data measurement. With this limitation, iterative transform algorithms have never been considered to be reliable enough. Our goal in this thesis is to understand the stagnation issues in iterative algorithms in general and work toward eliminating them so that the algorithms may become more resilient in order to achieve reliable solutions from practical far-field intensity data. In this quest, we offer a new parameter called 'complexity,' which provides a measure of the object's fluctuations and may be derived *a priori* from raw Fourier intensity data. We describe the stagnation artifacts in phase retrieval solutions using an entirely new viewpoint of complexity in order to analyze the behavior of solutions obtained by state-of-the-art algorithms. In particular, the complexity of estimated solution is compared with the desired complexity determined from the raw Fourier data. For instance, a higher value of complexity of the estimated solution, as compared to the desired complexity, is associated with the additional undesirable fluctuations or artifacts in the solution. With this information, we suggest a novel methodology of 'complexity-guided phase retrieval (CGPR)', which is a regularization scheme controlled by a complexity parameter that, when combined with standard algorithms, provides a high quality and reliable phase retrieval solution. Further, the performance of the CGPR algorithm was evaluated on the experimental X-ray coherent diffractive imaging data. This thesis provides fresh insights into the operation of standard iterative algorithms as well as new results on robust Fourier phase retrieval. We believe that the study presented in this thesis contributes a new understanding of phase retrieval

technique, thus improving the viability of existing methods for practical applications or devices.

सार

विज्ञान एवं इंजीनियरिंग के कई क्षेत्रों में यदि हम किसी वस्तु के बारे में कुछ समझना चाहते हैं, उसके लिए उससे जुड़ा फेज बहुत महत्वपूर्ण होता है। हालांकि वर्तमान में उपलब्ध फोटोडिटेक्टर केवल आयाम (एम्पलीट्यूड) सूचना रिकॉर्ड कर सकते हैं जबकि फेज सूचना को मापने में सक्षम नहीं हैं। फेज इमेजिंग का उद्देश्य वस्तु के खोये हुए फेज को उसके मापे गए आयाम से पुनर्प्राप्त करना है। फूरियर फेज रिट्रीवल एक महत्वपूर्ण फेज इमेजिंग तकनीकों में से एक है, जो की एक गैर-इंटरफेरोमेट्रिक तकनीक है, जिसमें किसी वस्तु के दूर-क्षेत्र या फ्रौनहोफर विवर्तन तीव्रता को मापा जाता है और फिर कम्प्यूटेशनल रूप से पुनरावृत्त विधियों (अल्गोरिथ्म्स) का उपयोग करके वस्तु का फेज पुनर्प्राप्त किया जाता है। चूँकि फूरियर फेज रिट्रीवल गैर-उत्तल और गैर-रैखिक है, इसलिए इसको हल करके एक अद्वितीय फेज खोजना चुनौतीपूर्ण है। फेज रिट्रीवल समस्या को हल करने के लिए आईट्रेटीव अल्गोरिथ्म्स लगभग आधी शताब्दी पूर्व से विकसित किए जा रहे हैं और ये उपयोगकर्ताओं के बीच एक लोकप्रिय विकल्प हैं। हालांकि, ये अल्गोरिथ्म्स एक रैंडम प्रारंभिक अनुमान से एकल-रन में उच्च-गुणवत्ता और विश्वसनीय समाधान (ऑब्जेक्ट रिकवरी) प्रदान करने में असमर्थ हैं। विशेषतः ये अल्गोरिथ्म्स फॉरिएर इंटेंसिटी डाटा से ऐसा फॉरिएर फेज प्राप्त करते हैं जिसके फलस्वरूप ऑब्जेक्ट रिकवरी में बहुत सारे अवांछित रूपरेखाएँ (आर्टिफैक्ट्स) भी आ जाती हैं जो की ऑब्जेक्ट ऑफ़ इंटरेस्ट का अध्ययन करने में समस्यात्मक होती हैं। इन कमियों के चलते, पुनरावृत्त रूपांतरण एल्गोरिदम (आईट्रेटीव ट्रांसफॉर्म अल्गोरिथ्म्स) को कभी भी पर्याप्त विश्वसनीय नहीं माना गया है और इसी वजह से आज फेज रिट्रीवल पे आधारित कोई रियल-टाइम डिवाइस उपलब्ध नहीं है। इस थीसिस में हमारा लक्ष्य पुनरावृत्त अल्गोरिथ्म्स की समस्याओं को समझना और उन कमियों को खत्म करने की दिशा में काम करना है ताकि विश्वसनीय फेज पुनर्प्राप्त किया जा सके। इस खोज में, हम 'कम्प्लेक्सिटी' नामक एक नया पैरामीटर प्रस्तावित करते हैं, जो किसी वस्तु की जटिलता का माप प्रदान करता है। कम्प्लेक्सिटी पैरामीटर की खासियत है की यह मापे गए फूरियर तीव्रता डेटा से प्राप्त किया जा सकता है। यही नहीं, इस पैरामीटर से हम किसी भी एल्गोरिदम द्वारा प्राप्त ऑब्जेक्ट रिकवरी का विश्लेषण भी कर सकते हैं। अतः हम एक पूरी तरह से नए दृष्टिकोण का उपयोग करके एक नवीन फेज रिट्रीवल अल्गोरिथम पद्धति को प्रस्तावित करते हैं जिसको हम `कम्प्लेक्सिटी-गाइडेड फेज रिट्रीवल (सीजीपीआर)' से नामित करते हैं। यह कम्प्लेक्सिटी पैरामीटर द्वारा नियंत्रित एक नियमितीकरण अल्गोरिथ्मिक योजना है, जो किसी भी मौजूदा एल्गोरिदम के साथ संयुक्त होने पर, एक उच्च गुणवत्ता और विश्वसनीय फेज पुनर्प्राप्त करती है। इसके अलावा, सीजीपीआर एल्गोरिथ्म के प्रदर्शन का मूल्यांकन प्रायोगिक एक्स-रे कोहेरेंट डिफ्रैक्शन इमेजिंग डेटा पर भी किया गया है। यह थीसिस आईट्रेटीव एल्गोरिदम के संचालन को समझने के साथ-साथ विश्वसनीय फूरियर फेज रिट्रीवल पर नए परिणाम प्रदर्शित करती है। इस थीसिस में प्रस्तुत कम्प्लेक्सिटी-गाइडेंस अध्ययन से फेज रिट्रीवल साहित्य में एक नई समझ का योग हुआ है, और इसकी उपयोगिता में कई व्यावहारिक अनुप्रयोग आधारित डिवाइस को सक्षम करने की क्षमता है।

Contents

List of Abbreviations

CDI	Coherent Diffraction Imaging
CGH	Computer Generated Hologram
CGPR	Complexity-guided Phase retrieval
CXI	Coherent X-ray Imaging
DM	Difference Map
ER	Error Reduction
FPM	Fourier Ptychographic Microscopy
GS	Gerchberg Saxton
HIO	Hybrid Input Output
NMSE	Normalized Mean Square Error
NRMSE	Normalized Root Mean Square Error
PSF	Point Spread Function
RAAR	Relaxed Averaged Alternating Reflections
SNR	Signal to Noise Ratio
SSIM	Structural Similarity Index Measure
TV	Total Variation

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