

AUTOMATIC MESH GENERATION, ERROR ESTIMATION AND ADAPTIVITY IN FINITE ELEMENT ANALYSIS OF METAL-FORMING PROCESSES

by

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submitted

in fulfilment of the requirements of the Degree of

DOCTOR OF PHILOSOPHY

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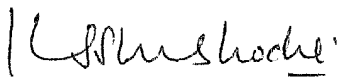
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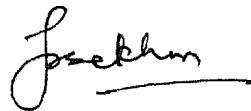
CERTIFICATE

This is to certify that the thesis entitled '***Automatic Mesh Generation, Error-Estimation and Adaptivity in Finite Element Analysis of Metal-Forming Processes***' being submitted by *Sh. Devinder Singh* to the **Indian Institute of Technology, Delhi** for the award of the Degree of Doctor of Philosophy is a record of bonafide research work carried by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard for the Doctor of Philosophy Degree.

The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree or diploma.



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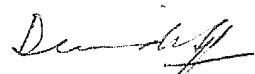
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ABSTRACT

With the advent of powerful personal computers, finite element simulation of metal forming processes has gained considerable prominence in industry. The simulation is required to be carried out automatically and with acceptable accuracy even as the domain keeps on continuously changing with processing time. The material undergoes very large plastic deformation and some of the elements in the mesh may suffer severe distortion. This could lead to inaccuracies and it may not even be possible to proceed with the degenerate mesh. In order to proceed with the analysis, a new, undistorted mesh is required to be automatically regenerated. Further, solution error tends to accumulate during the process of simulation. The novice user is not often capable of predicting error and controlling the mesh size. It is useful to estimate error and generate mesh of required density automatically to keep the error under control.

The present work is devoted to automatic finite element mesh generation, error estimation, mesh adaptivity and simulation of metal forming processes. An algorithm based on the advancing front technique has been proposed for automatic generation of a mesh comprising of triangular elements in which mesh density can be easily controlled. A scheme of combining triangular elements to generate a mesh of quadrilateral elements has been also been proposed. A computer program, named as **admesh**, has been developed to implement the above algorithm.

A finite element analysis code **form2d** has also been developed. It is based upon a flow formulation. Two types of material flow rules namely the rigid-plastic and rigid-visco plastic have been implemented. The elements types incorporated in

the code include 6-node triangular, and 4-node and 9-node quadrilateral elements. The incompressibility constraint has been applied through a penalty function that is evaluated by reduced integration. Frictional effects between the workpiece and tool have been accounted for. The evolving contact is updated through an automatic search procedure that projects all the nodes crossing the tool boundary on to the tool-work interface. The program takes care of temperature effects during simulation by performing coupled thermo-mechanical analysis.

An estimation technique for error in the finite element solution has also been proposed. It is based on the least squares fitting of displacement or velocity field over an elemental patch. The performance of proposed technique has been compared with that of the Zienkiewicz-Zhu recovery scheme by applying it to a test problem in incompressible elasticity using structured as well as unstructured meshes. The proposed scheme is well suited to adaptive mesh applications. Lastly, the robustness and applicability of the proposed automatic mesh generator, finite element analysis code and the error estimation technique have been demonstrated by simulating a few cases of non-steady and steady metal forming processes.

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