

SYSTEMS CONNECTIONS AND CATEGORIES

By

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CERTIFICATE

Certified that the research work "Systems Connections and Categories" by T. Karunakaran has been carried out under my supervision at the Indian Institute of Technology, Delhi, and that this work has not been submitted elsewhere for the award of a degree or diploma.

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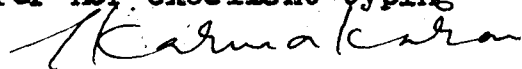
Professor R.E. Kalman brought to my attention the works of Professor Goguen, which further led me to become aware of the existing literature on categorical system theory, at a time when my research ambitions about a functorial foundation of networks were based mainly on my own intuitions. His optimistic comments, when I first mentioned about tolerance realisation to him during 1971 encouraged me to proceed further.

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ABSTRACT

There are two major aspects of the thesis: one devoted to developing an arrow theoretic basis for network related systems with a view to bringing their study into the domain of the currently emerging categorical system theory; and the other to the formulation of a model reduction based realisation theory applicable to tolerance machines.

Grouped under the theme 'Connection' are chapters two and three which mainly aim at modeling the Kirchhoff's relations of a network in terms of certain algebraic structures related to its underlying graph. Among other results obtained as byproducts, the following are worth mentioning: interpretation of primitive networks as universal and couniversal elements of certain categories, characterisation of cutsets as quotient objects and establishment of their 'exactness' properties in relation to a related set of circuit subobjects, and, the study of network interconnection and reconnection employing free products helpful in understanding the behaviour of Kirchhoff models under free product, under reconnection and under matrix representation.

The fourth chapter entitled functorial foundations of networks proposes a novel view point in modeling network concepts which consists in functorially endowing the Kirchhoff's relation models with the constituent relations.

In this process current and voltage concepts get modelled in terms of functors of covariant and contravariant nature respectively and dinatural transformations take the place of immittances. Based on the above framework we interpret the classical network formulations and primitive-to-network transformations, introduce notions like generalised primitive network, input object and output object and also deal with aspects of network solutions and functorial realisations. In the final sections we are also led to discuss about a subclass of general systems called 'disystems' to clarify the foundational aspects of the above developments.

The chapter entitled tolerance realisation theory introduces a realisation theory for automata whose output space has certain tolerance characteristics. The theory advances (nonunique) machine realisations with state spaces of reduced order (as compared to the Nerode theory based machines) and further permits specialisation to the Nerode realisation theory. Finally this theory applies to the linear system case also by use of a structured version of the tolerance condition and enables finite termination considerations analogous to the Nerode realisation theory to be conceived of.