SYSTEMS CONNECTIONS AND CATEGORIES

By

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A thesis submitted to the Indian Institute of Technology, Delhi for the award of the Degree of Doctor of Philosophy

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June 1975

CERTIFICATE

Certified that the research work "Systems Connections and Categories" by T. Karunakaran has been carried out under my supervision at the Indian Institute of Technology, Delhi, and that this work has not been submitted elsewhere for the award of a degree or diploma.

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ACKNOWLEDGEMENTS

My thesis supervisor Professor P.S. Satsangi has willingly accepted all sufferings in the process of helping me to achieve satisfaction in my complicated research ambitions. Finding the conventional words too inadequate to express my sense of gratitude I adopt the only meaingful alternative of making a resolve to make some tangible contribution at some period of my career which will further the cause of his mission based on the theme: "all social problems are system problems".

My grateful thanks are due to my friend Dr. A.K. Srivastava who acted as a source of information and counsel on matters related to categories and spent many of his precious hours in scrutinising my writings. His very presence here provided much encouragement and strength.

Professor R.E. Kalman brought to my attention the works of Professor Goguen, which further led me to become aware of the existing literature on categorical system theory, at a time when my research ambitions about a functorial foundation of networks were based mainly on my own intutions. His optimistic comments, when I first mentioned about tolerance realisation to him during 1971 encouraged me to proceed further.

I am grateful to Professor J.A. Goguen for his kind supply of many crucial reprints, his encouragement and for his constructive comments on a portion of the thesis.

I gratefully acknowledge the encouragements and helps (as well as knowledge related to system theory) received from Prof. S.C.Dutta Roy, Prof. A.K. Mahalanabis and Prof. V.S.Rajamani and also wish to record my indebtedness to Dr. S.N.Patnaik, Dr. J.B. Srivastava and Dr. W. Shukla who have taught me mathematics.

My sincere thanks go to the C.S.I.R., and I.I.T.,Delhi fo the financial support received during the tenure of my Ph.D. research and also wish to thank Professor M. Venkatraman of Madurai University, Mathematics Department and Professor C. Seshadri of T.I.F.R. School of Mathematics for the opportunities of teaching and learning I had while working at their centre during 1973-74. I am grateful to my seniors Dr.M.Ibrams and Dr. K.K. Biswas for various helps and also gratefully remember Professor K. Nagappan who was responsible for my initial interest in large scale systems.

Finally I thank Miss Neelam for her excellent typing of my thesis.

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CONTENTS

ABSTRACT

There are two major aspects of the thesis: one devoted to developing an arrow theoretic basis for network related systems with a view to bringing their study into the domain of the currently emerging categorical system theory; and the other to the formulation of a model reduction based realisation theory applicable to tolerance machines.

Grouped under the theme 'Connection' are chapters two and three which mainly aim at modeling the Kirchhoff's relations of a network in terms of certain algebraic structures related to its underlying graph. Among other results obtained as byproducts, the following are worth mentioning: interpretation of primitive networks as universal and couniversal elements of certain categories, characterisation of cutsets as quotient objects and establishment of their 'exactness' properties in relation to a related set of circuit subobjects, and, the study of network interconnection and reconnection employing free products helpful in understanding the behaviour of Kirchhoff models under free product, under reconnection and under matrix representation.

The fourth chapter entitled functorial foundations of networks proposes a novel view point in modeling network concepts which consists in functorially endowing the Kirchhoff's relation models with the constituent relations.

In this process current and voltage concepts get modelled in terms of functors of covariant and contravariant nature respectively and dinatural transformations take the place of immittances. Based on the above framework we interpret the classical network formulations and primitive-to-network transformations, introduce notions like generalised primitive network, input object and output object and also deal with aspects of network solutions and functorial realisations. In the final sections we are also led to discuss about a subclass of general systems called 'disystems' to clarify the foundational aspects of the above developments.

The chapter entitled tolerance realisation theory introduces a realisation theory for automata whose output space has certain tolerance characteristics. The theory advances (nonunique) machine realisations with state spaces of reduced order (as compared to the Nerode theory based machines) and further permits specialisation to the Nerode realisation theory. Finally this theory applies to the linear system case also by use of a structured version of the tolerance condition and enables finite termination considerations analogous to the Nerode realisation theory to be conceived of.