

STABILITY OF LAMINAR BOUNDARY LAYER FLOW
OVER A FLAT PLATE WITH A COMPLIANT SURFACE

D.S. ARORA

Department of Applied Mechanics
Indian Institute of Technology, Delhi
New Delhi

THESIS

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Indian Institute of Technology, Delhi
New Delhi

CERTIFICATE

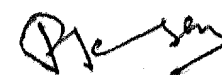
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(P.K. Sen)

Assistant Professor
Department of Applied Mechanics
Indian Institute of Technology,
New Delhi-110016 (India)

"Revised Thesis ~~and~~
submitted, and revision
approved by me."


15/9/87
(P. K. Sen)

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(D.S. ARORA)

ABSTRACT

The object of the present work is to study the effect of wall pliability on stabilization of boundary layer flow past a flat plate. A study of dolphins and subsequent experimental work, as reported by Kramer (1965), indicate that wall pliability could stabilize the flow past a flat plate.

The problem was examined theoretically and in detail by Benjamin (1964), Landahl (1962), Hans (1965), Landahl & Kaplan (1965), and Carpenter & Garrad (1985). They assumed the flat plate surface to comprise of a stabilizing coating, consisting of a visco-elastic or elasto-dynamic material, which deformed normal to the surface under the action of wall pressure. The problem was studied using the well-known small disturbance theory of Tollmien and Schlichting. The Orr-Sommerfeld equation was solved for a certain feasible range of surface properties of the stabilizing coating. It was reported that some of the rigid wall unstable modes could be stabilized at certain values of the surface properties. At the same time, these very values of the surface properties also tended to destabilize a certain other type of mode, which did not exist for the rigid wall case. It was therefore concluded that a stabilizing coating which deformed under normal pressure, could not lead to appreciable stabilization of the flow.

It was felt that the problem should be approached from a different angle. A kinematic model of the pliable surface was considered. The surface was allowed to deform normal to itself, under the action of perturbation pressure. The surface was allowed to oscillate with different amplitudes and phase angles, whilst keeping a fixed normalization for the disturbance stream-function at a fixed distance

from the flat plate surface.

It was found that as the magnitude of the stream-function $|\bar{\phi}_w|$ increased, the disturbance phase velocity c_r and the amplification factor c_i were affected. With the superimposed variation of phase angle θ , over $0 \leq \theta \leq 360^\circ$, the flow was destabilized over some part of the θ cycle and stabilized over the other part of the θ cycle. It was also revealing to find that the points of neutral stability obtained by previous theoreticians lay in a small region in the vicinity of $\theta = 180^\circ$.

As the amplitude of oscillation of the pliable wall was further increased, it was found that the original T-S (Tollmien-Schlichting) mode bifurcated into transitional modes which turned into a low-speed stable mode-class, and, a 'Resonance' mode-class. The latter was stable for part of the θ cycle only. Also one of the transitional modes, when made to approach low values of $|\bar{\phi}_w|$, was found to exhibit certain properties which were akin to the Kelvin-Helmholtz type of instability. As $\bar{\phi}_w \rightarrow 0$, it was seen that c_r also $\rightarrow 0$, i.e. standing waves were being formed. These waves were also unstable for part of the cycle of θ .

The physical realizability or otherwise of the various modes could be ascertained by using the kinematic model results to obtain back-calculated values of material properties of actual stabilizing coatings, and seeing whether these values of material properties were feasible and realistic. A pliable surface, modelled similarly as the one proposed by Landahl, was selected as all other complex surface models are reducible to this simple one. The back-calculated surface properties of this surface were obtained and plotted. This was done for several values of $|\bar{\phi}_w|$ and θ , and, for the different mode-classes.

A study of the plots confirmed that for stabilizing different mode-classes, the requirements for the surface properties of the stabilizing coating were of conflicting nature, and, it was difficult to find a single set of surface properties which would stabilize all the modes and thus cause the critical Reynolds number to increase by an appreciable amount. It is also desirable that more number of points in the $(\alpha-R)$ -plane be investigated, if other models for the stabilizing-coating are to be looked into. This has been left as a work for the future.

In the present work a sample of five representative points in the $(\alpha-R)$ -plane were examined. The kinematic model results showed that the behaviour of the various mode-classes at these different points are qualitatively similar, and believably so in the entire $(\alpha-R)$ -plane.

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