DISCRETE-TIME STOCHASTIC SLIDING MODE CONTROL USING FUNCTIONAL OBSERVATION

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by

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DEPARTMENT OF ELECTRICAL ENGINEERING

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CERTIFICATE

This is to certify that the thesis entitled **"Discrete-Time Stochastic Sliding Mode Control Using Functional Observation"**, submitted by **Satnesh Singh** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Electrical Engineering, is a record of the bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the award of the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

Dr. S. Janardhanan

Department of Electrical Engineering,

Indian Institute of Technology Delhi.

(Supervisor)

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Satnesh Singh

ABSTRACT

Sliding mode control (SMC) is one of the most efficient robust methods for controlling systems with uncertainty. SMC methods provide some advantages such as easy and straightforward realisation, quick response and low sensitivity to uncertainties in the model of the system and disturbances. The target of this thesis is to provide a comprehensive investigation of functional state estimation based SMC for discrete-time stochastic systems and propounds an idea of discrete-time stochastic SMC. SMC for discrete-time stochastic systems with bounded disturbances is proposed in this work. Subsequently, SMC is designed for the discrete-time stochastic system such that the states will lie within the specified band. Furthermore, this result has been extended for the incomplete state information. In that case, states are estimated by the Kalman filter approach and SMC is designed.

A functional observer based SMC is designed for discrete-time stochastic systems. The control signal is calculated by a functional observer method. In many cases, the disturbances in the system may not always satisfy the matching condition. Therefore, a functional observer based SMC is designed for discrete-time stochastic systems with unmatched uncertainty. A disturbance dependent based sliding surface method is proposed to ameliorate the effect of unmatched uncertainty in the stochastic system. Next, SMC design using functional state estimation is proposed for parametric uncertain discrete-time stochastic systems. A sufficient condition of stability is proposed based on Gershgorin disc theorem, which provides the estimate of the eigenvalues location of the matrix.

Most of engineering systems have an inherent time-delay such as sensor measurements and communication delays. This thesis also proposes SMC method for linear discrete-time delayed stochastic systems. Subsequently, the stability and convergence analysis of the proposed method are provided. Furthermore, SMC of a delayed stochastic system for incomplete state information has also been considered, where states are estimated by the Kalman filter approach. Moreover,

a functional observer based SMC method for the discrete-time delayed stochastic system is proposed. Therefore, SMC has been calculated by the functional observer approach. Finally, functional observer-based state feedback and SMC law are compared graphically as well as numerically.

Further, the previous result is extended for the case where parametric uncertainty are present in the system and the state delay matrix. Functional observer-based SMC is developed for uncertain state-delayed systems. Subsequently, the SMC method for discrete-time delayed stochastic systems is investigated. Finally, SMC has been calculated by using a functional observer approach. A sufficient condition of stability is proposed based on Gershgorin disc theorem, which provides the estimate of the eigenvalues location of the given matrix. Finally, in the last chapter, a summary of the thesis contributions is presented and future directions of research are identified.

Keywords: Sliding mode control, Functional observers, Time delay, Discrete-time systems, Stochastic systems.

सार

सिस्टम को नियंत्रित करने के लिए स्लाइडिंग मोड कंट्रोल (एसएससी) सबसे अधिक ई-रोगी मजबूत तरीकों में से एक है अनिश्चितता के साथ। एसएमसी विधियां कुछ फायदे प्रदान करती हैं जैसे कि आसान और सरल अहसास, त्वरित प्रतिक्रिया और सिस्टम के मॉडल में अनिश्चितताओं के प्रति कम संवेदनशीलता और गड़बड़ी। इस थीसिस का लक्ष्य एक व्यापक जांच प्रदान करना है असतत समय स्टोकेस्टिक प्रणालियों के लिए कार्यात्मक राज्य का अनुमान एसएमसी आधारित है और प्रॉप्स ए असतत समय स्टोकेस्टिक एसएमसी का विचार। बाउंड के साथ असतत समय स्टोकेस्टिक सिस्टम के लिए एसएमसी इस कार्य में गड़बड़ी प्रस्तावित है। इसके बाद, एसएमसी असतत समय के लिए डिज़ाइन किया गया है स्टोकेस्टिक प्रणाली ऐसी है कि राज्य विशिष्ट एड बैंड के भीतर स्थित होंगे। इसके अलावा, यह अपूर्ण राज्य सूचना के लिए परिणाम बढ़ाया गया है। उस स्थिति में, राज्यों का अनुमान है कलमन फ़िल्टर दृष्टिकोण और एसएमसी द्वारा डिज़ाइन किया गया है।

एक कार्यात्मक पर्यवेक्षक आधारित एसएमसी असतत समय स्टोकेस्टिक सिस्टम के लिए डिज़ाइन किया गया है। नियंत्रण संकेत एक कार्यात्मक पर्यवेक्षक विधि द्वारा गणना की जाती है। कई मामलों में, गड़बड़ी सिस्टम में हमेशा मिलान स्थिति को संतुष्ट नहीं किया जा सकता है। इसलिए, एक कार्यात्मक पर्यवेक्षक आधारित एसएमसी को बेजोड़ अनिश्चितता के साथ असतत समय स्टोकेस्टिक सिस्टम के लिए डिज़ाइन किया गया है। ए अशांति पर आधारित स्लाइडिंग सतह विधि के प्रभाव को संशोधित करने का प्रस्ताव है स्टोकेस्टिक प्रणाली में बेजोड़ अनिश्चितता। अगला, कार्यात्मक राज्य का उपयोग करके एसएमसी डिजाइन अनुमान पैरामीट्रिक अनिश्चित असतत समय स्टोकेस्टिक सिस्टम के लिए प्रस्तावित है। एक पर्याप्त Gershgorin डिस्क प्रमेय के आधार पर स्थिरता की स्थिति प्रस्तावित है, जो अनुमान प्रदान करती है मैट्रिक्स के आइगेन्वॅल्यूस स्थान।

अधिकांश इंजीनियरिंग प्रणालियों में एक अंतर्निहित समय-देरी होती है जैसे सेंसर माप और संचार में देरी। यह थीसिस भी रैखिक असतत समय देरी के लिए एसएमसी विधि का प्रस्ताव है स्टोकेस्टिक सिस्टम। इसके बाद, प्रस्तावित विधि की स्थिरता और अभिसरण विश्लेषण दिए गए है। इसके अलावा, अपूर्ण राज्य सूचना के लिए एक विलंबित स्टोचस्टिक प्रणाली का एसएमसी यह भी माना जाता है, जहां राज्यों का अनुमान कलमन लेटर दृष्टिकोण से है। इसके अलावा, असतत समय देरी वाले स्टोकेस्टिक सिस्टम के लिए एक कार्यात्मक पर्यवेक्षक आधारित एसएमसी विधि है का प्रस्ताव रखा। इसलिए, एसएमसी की गणना कार्यात्मक पर्यवेक्षक दृष्टिकोण द्वारा की गई है। आखिरकार, कार्यात्मक पर्यवेक्षक-आधारित राज्य प्रतिक्रिया और एसएमसी कानून की तुलना रेखांकन के साथ-साथ की जाती है संख्यानुसार।

इसके अलावा, पिछले परिणाम को उस मामले के लिए बढ़ाया जाता है जहां सिस्टम में पैरामीट्रिक अनिश्चितता और राज्य विलंब मैट्रिक्स मौजूद हैं। कार्यात्मक प्रेक्षक आधारित एसएमसी अनिश्चित के लिए विकसित किया गया है राज्य-विलंबित सिस्टम। इसके बाद, असतत समय के लिए एसएमसी विधि स्टोकेस्टिक प्रणालियों में देरी हुई जांच की जाती है। अंत में, एक कार्यात्मक पर्यवेक्षक दृष्टिकोण का उपयोग करके एसएमसी की गणना की गई है। Gershgorin डिस्क प्रमेय के आधार पर स्थिरता की पर्याप्त स्थिति प्रस्तावित है, जो प्रदान करती है दिए गए मैट्रिक्स के आइगेन्वॅल्यूस स्थान का अनुमान। अंत में, आखिरी अध्याय में, ए थीसिस योगदान का सारांश प्रस्तुत किया गया है और अनुसंधान की भविष्य की दिशाएं आइडी एड हैं।
कीवर्ड: स्लाइडिंग मोड नियंत्रण, कार्यात्मक पर्यवेक्षक, समय की देरी, असतत समय प्रणाली, स्टोकेस्टिक सिस्टम।

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Nomenclature

Acronyms	**Description**
CSMC	Continuous-time Sliding Mode Control
DSM	Discrete-time Sliding Mode
DSMC	Discrete-time Sliding Mode Control
QSM	Quasi Sliding Mode
QSMB	Quasi Sliding Mode Band
KF	Kalman Filter
LTI	Linear Time Invariant
LFO	Linear Functional Observers
LQR	Linear Quadratic Regulator
LMI	Linear Matrix Inequality
SFC	State Feedback Control
SMB	Sliding Mode Band
SMC	Sliding Mode Control
DARE	Discrete-time Algebraic Riccati Equation
a.s.	Almost sure
p.d.s.	Positive definite symmetric
VSS	Variable Structure Systems
VSC	Variable Structure Control

Continued from previous page

Acronyms	Description
VSCS	Variable Structure Control Systems

List of Symbols	Description
$(.)^T$	General notation for the matrix transpose operation
$\mathbb{R}$	The field of real numbers
$\mathbb{R}^n$	The n-dimensional real vector space
$\mathbb{Z}$	The set of integer numbers
$\otimes$	Kronecker product
A	State matrix in discrete-time system
A_d	State matrix in discrete-time system with time delay
C	Output matrix in discrete-time system
B	Input matrix of control in discrete-time system
Γ	Process noise matrix in discrete-time system
F	Unknown input matrix of the system
G	Measurement noise matrix in discrete-time system
d	The disturbance vector effect on the sliding function
d_0	The mean of disturbance bounds d_u and d_l
d_l	Lower bound of the disturbance d
d_u	Upper bound of the disturbance d
P	Probability measure
Ω	Sample space
$\mathscr{F}$	Set of events

List of Symbols	Description
I_n	An $n \times n$ identity matrix
0_n	An $n \times n$ zero matrix
u	Control input
θ	Regulating the approaching speed
$\mathbb{E}(.)$	Expectation operator
$\mathbb{E}\{.\|.\}$	Conditional expectation
k	Variable denoting time instant
k_x	Constant known delay in state matrix
m	Number of inputs in system
n	Number of states in system
p	Number of outputs of system
w	Vector of process noise
v	Vector of measurement noise
Q	Matrix of process noise co-variance
R	Matrix of measurement noise co-variance
$\|\|.\|\|$	Euclidean norm, the norm of a vector
q	Order of functional observer
s	Sliding function
$S_{\mu c}$	Sliding mode band
$\tau > 0$	Sampling time
κ, ψ	Tuning parameters
$\forall$	For all
$\square$	Designating the completion of proof
$\rightarrow$	Tends to

List of Symbols	**Description**
$\triangleq$	Defined as
$\subset$	Proper subset of
$\cap$	Intersection
$\in$	Belongs to
$\mathbb{C}$	Complex plane
δ	Performance index
$\rho(X)$	Range space of the matrix X
x	State of a dynamical system
$\hat{x}$	Estimated system state of a dynamical system
y	System output of a dynamical system
Υ	Kalman gain
c	Sliding function parameter
K	Sliding function gain matrix
V	Lyapunov function
U	General notation for an $n \times n$ transformation matrix
L	Functional gain matrix
M, J, H, E	Unknown functional observer matrices
M_d, J_d, E_d	Unknown functional observer matrices with time delay