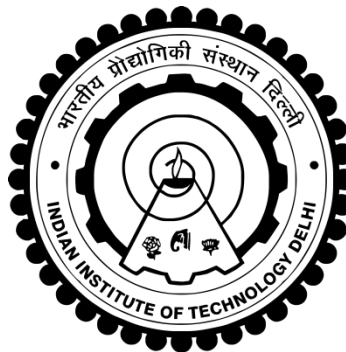


**HYPERVISCOELASTIC CONSTITUTIVE MODELLING
AND CRACK PROPAGATION BEHAVIOR
OF SOLID PROPELLANT**

NOMESH KUMAR



**DEPARTMENT OF APPLIED MECHANICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
NOVEMBER 2018**

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HYPERVISCOELASTIC CONSTITUTIVE MODELLING AND CRACK PROPAGATION BEHAVIOR OF SOLID PROPELLANT

by

NOMESH KUMAR

DEPARTMENT OF APPLIED MECHANICS

Submitted

In fulfilment of the requirements of the Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

NOVEMBER 2018

Certificate

This is to certify that the thesis entitled "Hyperviscoelastic Constitutive Modelling and Crack Propagation Behavior of Solid Propellant" being submitted by Mr. Nomesh Kumar to the Indian Institute of Technology Delhi for the award of degree of Doctor of Philosophy in Applied Mechanics is a record of original, bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

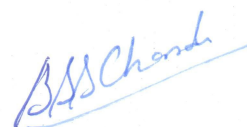
The results contained in the thesis have not been submitted in part or in full, to any other university or institute for the award of any degree or diploma.

Date: 13 / 08 /2018

Place: New Delhi

(B. P. Patel)

Professor, Department of Applied Mechanics
Indian Institute of Technology Delhi
New Delhi, 100016, India



(B. S. Subhashchandran)

Distinguished Scientist
Defence Research and Development Laboratory
Defence Research and Development Organisation
Hyderabad, 500058, India

Acknowledgment

I would like to express my gratitude to my supervisor, Prof. B. P. Patel, Department of Applied Mechanics, IIT- Delhi and Dr. B. S. Subhashchandran, Distinguished Scientist, Defence Research and Development Laboratory, DRDO, for their valuable guidance that helped me to develop the research skills, and especially for training me to be a better professional in thinking, executing, writing and presenting my work. I am indebted to their excellent guidance, motivation and support during the entire course of this program. I am also thankful to the faculty and staff of IIT- Delhi for their help and support.

I thank Prof. Santosh Kapuria, Prof. S. K. Roy, Prof. J. T. Shahu and Prof. Puneet Mahajan, the member of my Research Committee for their valuable suggestions. I am also thankful all the faculty members of the department for their support in accomplishing the present research work at IIT- Delhi.

I am also thankful to Dr. V. Venkateshwara Rao and Shri Papa Rao, Outstanding Scientists, Advanced Systems Laboratory, who encourages and supports me for accomplishing the present task. Their motivation and support at different occasions were valuable. I am also thankful to Shri Deo Kumar Verma, Shri R. S. Pail, Dr. K. Veerbhrahamam and Shri Umashankar, Scientists, DRDO for their support to carry out experiments.

I am also grateful to my beloved wife Sonalika Singh and sons Naman Pratap Karwal and Apoorva Pratap Karwal for their patience, motivation and support. I would like to express my deep sense of gratitude to my parents Shri Lal Singh and Smt. Shakuntla Verma for their motivation, support and blessing.

Nomesh Kumar

Abstract

Solid propellants are particulate composite materials consisting of high volume fraction of solid particles randomly suspended in a Hydroxyl Terminated Poly-butadiene (HTPB) based rubber matrix. The strong influence of high fraction of solid particles on mechanical properties is well known and is considered in formulating the constitutive relations. Mechanical behavior of solid propellant grains is studied from a phenomenological and micromechanical point of view based on the available literature and tests data and it can be concluded that the existing solid propellant constitutive models require a number of phenomenological constants and functions to describe the effect of damage and compressibility. Further, to the best of the author's knowledge, the combined effect of compressibility, damage and age hardening has not been studied. The objective of this study is to develop the constitutive model with least number of material constants to describe the compressibility, damage and age hardening effects simultaneously. It may be noted that the Mullins effect due to the highly non-uniform stress and strain gradient at particle matrix interface, micromechanical damage, nonlinearity in the stress- strain behavior and change of the bulk behavior from incompressible to compressible are important for accurate modelling of solid propellants.

The phenomenological model based upon continuum mechanics and micromechanism inspired model based on the statistical mechanics are formulated based on Simo and Bergstrom-Boyce models, respectively. Both the models represent strain rate, large deformation/large strain, thermo-rheological behavior, stress softening due to microstructural damage, compressibility and chemical age hardening. The damage/Pseudoelasticity is incorporated based on macroscopic continuum theory within the framework of irreversible thermodynamics. The nonlinear variation of bulk modulus with hydrostatic pressure is incorporated in the volumetric part. Due to chemical reaction, mechanical properties of solid propellant degrade with time and are incorporated based on Arrhenius equation. The proposed formulations represent entire mechanical behavior of solid propellants and validated with uniaxial tensile experimental data.

The phenomenological model is formulated by combining compressible Mooney- Rivlin hyperelastic strain energy density function with the standard model of viscoelasticity. To model the compressibility, the dilatational strain energy is taken as the hyperbolic function of the determinant of deformation gradient. The stress softening phenomenon during cyclic loading

(Mullins effect) due to microstructural damage is described by an exponential function of the current magnitude of intensity of strain and its previous maximum value. The variation of material properties with time are incorporated based upon experimental studied using isothermal accelerated ageing technique.

Micromechanism inspired Hyperviscoelastic constitutive model is formulating by combining both continuum formulation of hyperelasticity and statistical formulation of viscoelasticity. Compressibility is modelled by considering dilatational part of strain energy density as quadratic and quartic function of the dilation. The stress softening/Pseudoelasticity during cyclic loading (Mullins effect) due to microstructural damage is described by polynomial function of the current strain energy density and its previous maximum value.

The mechanical and fracture behavior of solid propellants at different strain rates and temperatures as well as the effect of age hardening on mechanical response and fracture behavior have been studied through experimentation on uniaxial tensile specimen and Compact Tension (CT) fracture specimen. Moreover the mechanical and fracture behavior has been studied with time through accelerated ageing technique. The Fracture toughness of solid propellant is determined at different strain rates and temperatures for different ageing time periods. During test, it was observed that the crack propagated in sharp - blunting fashion in unaged propellant and continuous sharp fashion in aged propellant. It was found that the fracture toughness increases with strain rate and decreases with ageing time. The fatigue experiments are carried out in displacement control mode. The crack length corresponding to maximum load of each cycle is determined and crack propagation law is proposed as per experimental results.

Dynamic structural integrity during ignition transient period is one of the important design criteria for solid rocket motors. During ignition transients, head end and nozzle end domes of motor case are exposed to pressure rise directly and not through the propellant grain. This leads to the greater stresses at the insulation /propellant interface at bond termination point resulting in the debonding of the propellant/ insulation interface. Dynamic structural integrity analysis of solid propellant grain with metallic casing is carried out considering ignition transient pressure loading using compressible viscohyperelastic constitutive model. The compressibility is incorporated through the volumetric strain energy terms.

During service life, a solid propellant rocket motor is stored in different stress inducing environments and is subjected to variable thermal stresses (thermal fatigue). The grain material

degrades due to thermally induced age hardening and cumulative damage leading to the decrease in the propellant load carrying capacity. Further, the material properties are statistically distributed. A probabilistic approach in conjunction with the non-linear viscoelastic analysis is employed incorporating test data (stress capacity and strain capability), age hardening trends and variation of input parameters to estimate the service life of a solid propellant rocket motor. The service life of the solid propellant rocket motor is assumed to be terminated when the progressive probability of failure which increases with the elapsed time exceeds a predetermined value.

सार

सॉलिड प्रोपेलेंट्स कणों की समग्र सामग्रियां हैं जिनमें ठोस कणों के उच्च मात्रा वाले अंश होते हैं जो हाइड्रोक्साइल टर्मिनेटेड पॉली-बटाडियन (एचटीपीबी) आधारित रबड़ मैट्रिक्स में यादृच्छिक रूप से निलंबित होते हैं। यांत्रिक गुणों पर ठोस कणों के उच्च अंश का मजबूत प्रभाव अच्छी तरह से जाना जाता है और इसे गठबंधन संबंध बनाने में माना जाता है। ठोस प्रणोदक अनाज के यांत्रिक व्यवहार का अध्ययन उपलब्ध साहित्य और परीक्षण डेटा के आधार पर एक घटनात्मक और सूक्ष्मदर्शी दृष्टिकोण से किया जाता है और यह निष्कर्ष निकाला जा सकता है कि मौजूदा ठोस प्रोपेलेंट गठित मॉडल को क्षति के प्रभाव का वर्णन करने के लिए कई घटनात्मक स्थिरांक और कार्यों की आवश्यकता होती है और संपीड़न। इसके अलावा, लेखक के सर्वोत्तम ज्ञान के लिए, संपीड़न, क्षति और आयु सख्त होने का संयुक्त प्रभाव अध्ययन नहीं किया गया है। इस अध्ययन का उद्देश्य संपीड़न, क्षति और उम्र सख्त प्रभावों का वर्णन करने के लिए कम से कम सामग्री स्थिरांक के साथ गठित मॉडल विकसित करना है। यह ध्यान दिया जा सकता है कि कूल मैट्रिक्स इंटरफेस पर अत्यधिक गैर-वर्दी तनाव और तनाव ढाल के कारण मुलिन्स प्रभाव, तनाव-तनाव व्यवहार में अनावश्यकता और संकुचित करने के लिए असम्पीडित से थोक व्यवहार में परिवर्तन सटीक मॉडलिंग के लिए महत्वपूर्ण हैं ठोस प्रणोदक।

सांख्यिकीय यांत्रिकी के आधार पर निरंतर यांत्रिकी और माइक्रोमैकेनिज्म प्रेरित मॉडल पर आधारित घटनात्मक मॉडल क्रमशः सिमो और बर्गस्ट्रॉम-बॉयस मॉडल के आधार पर तैयार किए जाते हैं। दोनों मॉडल तनाव दर, बड़े विरूपण / बड़े तनाव, थर्मो-रियोलॉजिकल व्यवहार, सूक्ष्म संरचनात्मक क्षति, संपीड़न और रासायनिक आयु सख्त होने के कारण तनाव नरम होने का प्रतिनिधित्व करते हैं। क्षति / Psuedoelasticity अपरिवर्तनीय थर्मोडायनामिक्स के ढांचे के भीतर मैक्रोस्कोपिक निरंतर सिद्धांत के आधार पर शामिल किया गया है। हाइड्रोस्टैटिक दबाव के साथ थोक मॉड्यूलस की nonlinear भिन्नता volumetric भाग में शामिल किया गया है। रासायनिक प्रतिक्रिया के कारण, ठोस प्रोपेलेंट के यांत्रिक गुण समय के साथ घटते हैं और एरेनियस समीकरण के आधार पर शामिल किए जाते हैं। प्रस्तावित फॉर्मूलेशन ठोस प्रणोदकों के पूरे यांत्रिक व्यवहार का प्रतिनिधित्व करते हैं और अनियमित तन्यता प्रयोगात्मक डेटा के साथ मान्य होते हैं।

घटनात्मक मॉडल को विस्कोलेस्टेंसी के मानक मॉडल के साथ संपीड़ित मूनी-रिविनिन हाइपरैलैस्टिक तनाव ऊर्जा घनत्व समारोह के संयोजन द्वारा तैयार किया जाता है। संपीड़न मॉडल करने के लिए, विचलनशील तनाव ऊर्जा विरूपण ढाल के निर्धारक के हाइपरबॉलिक फ़ंक्शन के रूप में ली जाती है। माइक्रोस्ट्रक्चरल क्षति के कारण चक्रीय लोडिंग (मुलिन्स प्रभाव) के दौरान तनाव नरम घटना का वर्णन तनाव की तीव्रता और उसके पिछले अधिकतम मूल्य की वर्तमान परिमाण के एक घातीय कार्य द्वारा किया गया है। समय के साथ भौतिक गुणों की विविधता इसोथर्मल त्वरित वृद्धावस्था तकनीक का उपयोग करके प्रयोगात्मक अध्ययन के आधार पर शामिल की जाती है।

माइक्रोमैकेनिज्म प्रेरित हाइपरविस्कोलास्टिक गठित मॉडल हाइपरैलैस्टिकिटी और विस्कोलोस्टिकिटी के सांख्यिकीय फॉर्मूलेशन के निरंतर फॉर्मूलेशन दोनों को जोड़कर तैयार कर रहा है। संपीड़न को तनाव ऊर्जा घनत्व के dilatational भाग पर विचार करके मॉडलिंग किया जाता है जैसे कि फैलाव के चौकोर और क्वार्टिक फ़ंक्शन। माइक्रोस्ट्रक्चरल क्षति के कारण चक्रीय लोडिंग (मुलिन्स प्रभाव) के दौरान तनाव

नरम / Psuedoelasticity वर्तमान तनाव ऊर्जा घनत्व और इसके पिछले अधिकतम मूल्य के बहुपद कार्य द्वारा वर्णित है।

विभिन्न तनाव दरों और तापमान पर ठोस प्रणोदकों के यांत्रिक और फ्रैक्चर व्यवहार के साथ-साथ यांत्रिक प्रतिक्रिया और फ्रैक्चर व्यवहार पर उम्र बढ़ने के प्रभाव का अध्ययन अनियमित तन्यता नमूना और कॉम्पैक्ट तनाव (सीटी) फ्रैक्चर नमूना पर प्रयोग के माध्यम से किया गया है। इसके अलावा यांत्रिक और फ्रैक्चर व्यवहार का अध्ययन तेजी से बढ़ने वाली तकनीक के माध्यम से किया गया है। ठोस प्रोपेलेंट की फ्रैक्चर क्रूरता अलग-अलग उम्र बढ़ने की अवधि के लिए अलग-अलग तनाव दरों और तापमान पर निर्धारित होती है। परीक्षण के दौरान, यह देखा गया था कि क्रेक उग्र प्रणोदक में तेज चमकदार फैशन और वृद्ध प्रणोदक में निरंतर तेज फैशन में फैला हुआ है। यह पाया गया कि फ्रैक्चर क्रूरता तनाव दर के साथ बढ़ जाती है और वृद्धावस्था के समय में कमी आती है। विस्थापन नियंत्रण मोड में थकान प्रयोग किए जाते हैं। प्रत्येक चक्र के अधिकतम भार से संबंधित दरार लंबाई निर्धारित होती है और प्रयोगात्मक परिणामों के अनुसार प्रकोप कानून को क्रेक किया जाता है।

इग्निशन क्षणिक अवधि के दौरान गतिशील संरचनात्मक अखंडता ठोस रॉकेट मोटर्स के लिए महत्वपूर्ण डिजाइन मानदंडों में से एक है। इग्निशन ट्रांजिस्टर के दौरान, मोटर केस के हेड एंड और नोजल एंड डोम्स सीधे प्रेशरेंट अनाज के माध्यम से दबाव वृद्धि के संपर्क में आते हैं। इससे बॉन्ड टर्मिनेशन पॉइंट पर इन्सुलेशन / प्रोपेलेंट इंटरफ़ेस पर अधिक तनाव होता है जिसके परिणामस्वरूप प्रणोदक / इन्सुलेशन इंटरफ़ेस की बहस होती है। धातु के आवरण के साथ ठोस प्रणोदक अनाज के गतिशील संरचनात्मक अखंडता विश्लेषण संपीड़ित viscohyperelastic गठित मॉडल का उपयोग कर इग्निशन क्षणिक दबाव लोडिंग पर विचार किया जाता है। संपीड़न वोल्यूमेट्रिक तनाव ऊर्जा शर्तों के माध्यम से शामिल किया गया है।

सेवा जीवन के दौरान, एक ठोस प्रोपेलेंट रॉकेट मोटर विभिन्न तनाव प्रेरित वातावरण में संग्रहीत होती है और परिवर्तनीय थर्मल तनाव (थर्मल थकान) के अधीन होती है। थर्मलली प्रेरित उम्र सख्त होने और संचयी क्षति के कारण अनाज सामग्री घट जाती है जिससे प्रोपेलेंट लोड ले जाने की क्षमता में कमी आती है। इसके अलावा, भौतिक गुण सांख्यिकीय रूप से वितरित किए जाते हैं। गैर रेखीय viscoelastic विश्लेषण के संयोजन के साथ एक संभाव्य दृष्टिकोण एक ठोस प्रोपेलेंट रॉकेट मोटर के सेवा जीवन का अनुमान लगाने के लिए परीक्षण डेटा (तनाव क्षमता और तनाव क्षमता), उम्र सख्त प्रवृत्तियों और इनपुट पैरामीटर की विविधता शामिल है। ठोस प्रणोदक रॉकेट मोटर की सेवा जीवन को समाप्त कर दिया जाता है जब विफलता की प्रगतिशील संभावना जो पूर्व निर्धारित समय के साथ बढ़ जाती है, पूर्व निर्धारित मान से अधिक हो जाती है।

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Nomenclature

a_T	WLF shift factor (Time temperature shift factor)
A_1, A_2, A_3	fracture toughness constants
A^e, B^e	bulk modulus constants for equilibrium part
A^{eT}, B^{eT}	bulk modulus constants for time dependent part
B_1, B_2, B_3	crack growth rate constants
B	ratio of activation energy to gas constants
b & d	damage constants
C_{10}, C_{01}, C_{11}	Mooney Rivlin constants for equilibrium part
C_{10}, C_{01}, C_{11}	Mooney Rivlin constants for equilibrium part
$C_{10}^{eT}, C_{01}^{eT}, C_{11}^{eT}$	Mooney Rivlin constants for time dependent part
$c_2, m, \hat{\tau}, \tau_f, b$ and γ_o	constants for time dependent part
$\mathbf{C}$	right Cauchy-Green tensor
$\bar{\mathbf{C}}$	distortional part of right Cauchy-Green tensor
CTE	coefficient of thermal expansion
$\mathbf{E}$	Lagrangian strain tensor
$\mathbf{E}^e$	elastic part of Lagrangian strain tensor
$\mathbf{E}_v$	viscoelastic part of Lagrangian strain tensor
$\mathbf{E}^{eT}$	time dependent elastic part of Lagrangian strain tensor
$\dot{\mathbf{E}}_v$	viscoelastic strain rate
J	volumetric ratio
J^e	equilibrium part of volumetric ratio
J^e	time dependent part of volumetric ratio
$\mathbf{F}$	deformation gradient tensor
$\bar{\mathbf{F}}$	disortional part of deformation gradient tensor
$\bar{\mathbf{F}}^e$	elastic disortional part of deformation gradient tensor
$\bar{\mathbf{F}}^{eT}$	time dependent elastic disortional part of deformation gradient tensor
$\bar{\mathbf{F}}^v$	viscoelastic disortional part of deformation gradient tensor
$\mathbf{L}^T$	equilibrium elastic part of velocity gradient tensor
$\mathbf{L}^{eT}$	time dependent elastic part of velocity gradient tensor
G_∞	Equilibrium shear modulus
G_0	Glassy shear modulus
G_i	Pony series coefficient of shear modulus
$\bar{\mathbf{H}}_i^n$	viscoelastic part of second Piola- Kirchhoff stress tensor

K_1, K_2	volumetric strain energy constants
m	current magnitude of strain intensity
$M \& N$	temperature shift function constants
P^*	load at fracture
p	hydrostatic pressure
$\mathbf{S}$	second Piola- Kirchhoff stress tensor
$\mathbf{S}^d$	deviatoric part of the second Piola- Kirchhoff stress tensor
$\bar{\mathbf{S}}_\infty^n$	elastic part of second Piola- Kirchhoff stress tensor
SFT	stress free temperature
ST	storage temperature
T	Temperature
t	time in sec
Δt	time increment
u^*	displacements at fracture
W	total strain energy density
α	normalized shear moduli
β_T	Arrhenius parameter
β	compressibility
ξ	reduced time
σ	axial stress
$\sigma_1, \sigma_2, \sigma_3$	principal stress
ψ	Helmholtz strain energy density
ψ_m	ψ_m = maximum Helmholtz strain energy density during cyclic loading