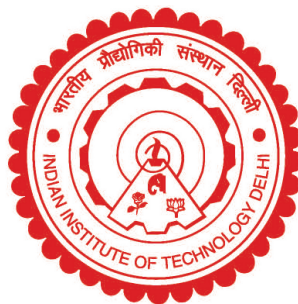


**A STOCHASTIC APPROACH
TO DELINEATE DOWNWARD RATING
MOMENTUM AND CLUSTERING PHENOMENON
IN CREDIT RISK**

PUNEET PASRICHA



**DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
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A STOCHASTIC APPROACH
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MOMENTUM AND CLUSTERING PHENOMENON
IN CREDIT RISK

by

Puneet Pasricha

Department of Mathematics

submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



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Dedicated To My Family

Certificate

This is to certify that the thesis entitled “**A Stochastic Approach to Delineate Downward Rating Momentum and Clustering Phenomenon in Credit Risk**” submitted by **Mr. Puneet Pasricha** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bonafide research work carried out by him under my supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Abstract

Credit risk is the risk of financial losses due to unexpected changes in the credit quality of a counterparty in a financial agreement. It is one of the significant risk faced by any financial institution. The critical aspect in modeling the credit risk is to predict if and when a default event is likely to occur. The increasing interest in modeling the credit risk is primarily a result of the increase in the off-balance-sheet derivatives, securitized loans and regulatory concerns. Also, the sup-prime crisis has considerably stressed the significance of having reliable measures of credit risk, in particular, the probability of default. In response, academicians and practitioners alike have developed new and more sophisticated models to measure the growing credit risk.

Broadly, there are two categories of credit risk models in the literature: the firm value models (also known as structural models) and the intensity based models (also known as reduced form models). The class of structural models is the oldest class of credit risk models. The structural models are more grounded in economic fundamentals and present an explicit mechanism of the default in terms of the relation between liabilities and assets. In this class of models, the firm is said to be in default if its value falls below the default barrier representing the liabilities. On the other hand, reduced form models do not specify the actual mechanism of default but model it as a non-negative random variable whose distribution depends on the economic factors.

This thesis aims to propose and analyze various stochastic models of the credit risk at an individual as well as at portfolio level to delineate the empirical characteristics of the credit rating dynamics and default events such as the downward rating momentum, the contagion effects, etc. The credit rating dynamics have several characteristics such as the downward rating momentum, the ageing effect, the dependence on the time of evaluation and the duration effect. We propose a time non-homogeneous Markov regenerative process based credit rating model to capture these characteristics and to overcome the limitations of the models in the literature.

The sub-prime crisis of 2007-08, which led to the collapse of some major financial institutions such as Lehman Brothers, has emphasized the significance of contagion risk in the financial market. The contagion risk is expected as the firms do not run in isolation and hence there exists a dependence among the firms due to common economic factors or direct business links. Therefore, this thesis aims to propose a contagion process, in a multivariate framework, that is capable of capturing the contagion effects. In the same spirit, we extend the contagion process to a regime-switching framework to incorporate the dynamic behavior of macro-economy, motivated by the fact that the prices of the financial securities are affected significantly by the state of the economy which itself is subjected to various regimes. Further, this thesis studies the utilization of these models to the pricing of credit portfolio derivatives. Continuing with the same theme, we proposes a Hawkes jump-diffusion structural credit risk model to capture the phenomenon of jump clustering in the asset prices that serve as proxies for the firm values.

Finally, this thesis attempts to propose a Markov chain credit rating model that includes the information present in the structural variables governing the health of a firm. The proposed model is a hybrid model designed to have the advantage of both a structural model in the sense that it gives a realistic description of default.

सार

एक वित्तीय समझौते में प्रतिपक्ष की क्रेडिट गुणवत्ता में अप्रत्याशित परिवर्तनों के कारण क्रेडिट जोखिम वित्तीय नुकसान का खतरा है। यह किसी भी वित्तीय संस्थान द्वारा सामना किए जाने वाले महत्वपूर्ण जोखिम में से एक है। क्रेडिट जोखिम को मॉडलिंग में महत्वपूर्ण पहलू भविष्यवाणी करना है कि डिफॉल्ट घटना होने पर और कब होने की संभावना है। क्रेडिट जोखिम को मॉडलिंग में बढ़ती दिलचस्पी मुख्य रूप से ऑफ-बैलेंस शीट डेरिवेटिव्स, सुरक्षित ऋण और नियामक चिंताओं में वृद्धि का परिणाम है। इसके अलावा, सुपर-प्राइम संकट ने क्रेडिट जोखिम के विश्वसनीय उपायों, विशेष रूप से, डिफॉल्ट की संभावना होने के महत्व पर जोर दिया है। जवाब में, शिक्षाविदों और चिकित्सकों ने समान रूप से बढ़ते क्रेडिट जोखिम को मापने के लिए नए और अधिक परिष्कृत मॉडल विकसित किए हैं।

व्यापक रूप से, साहित्य में क्रेडिट जोखिम मॉडल की दो श्रेणियां हैं: फर्म वैल्यू मॉडल (जिसे संरचनात्मक मॉडल भी कहा जाता है) और तीव्रता आधारित मॉडल (जिसे कम फॉर्म मॉडल भी कहा जाता है)। संरचनात्मक मॉडल की कक्षा क्रेडिट जोखिम मॉडल की सबसे पुरानी श्रेणी है। संरचनात्मक मॉडल आर्थिक मौलिक सिद्धांतों में अधिक आधार पर हैं और देनदारियों और परिसंपत्तियों के बीच संबंधों के संदर्भ में डिफॉल्ट की स्पष्ट प्रणाली प्रस्तुत करते हैं। मॉडल के इस वर्ग में, फर्म को डिफॉल्ट रूप से कहा जाता है यदि इसका मान देनदारियों का प्रतिनिधित्व करने वाले डिफॉल्ट बाधा से नीचे आता है। दूसरी तरफ, कम फॉर्म मॉडल डिफॉल्ट रूप से वास्तविक तंत्र निर्दिष्ट नहीं करते हैं लेकिन इसे गैर-ऋणात्मक यादृच्छिक चर के रूप में मॉडल करते हैं जिसका वितरण आर्थिक कारकों पर निर्भर करता है।

इस थीसिस का लक्ष्य किसी व्यक्ति के साथ-साथ पोर्टफोलियो स्तर पर क्रेडिट जोखिम गतिशीलता की अनुभवजन्य विशेषताओं और डाउनवर्ड रेटिंग गति, संक्रमित प्रभाव इत्यादि जैसी डिफॉल्ट घटनाओं को चित्रित करने के लिए क्रेडिट जोखिम के विभिन्न स्टोकास्टिक मॉडल का प्रस्ताव और विश्लेषण करना है। क्रेडिट रेटिंग गतिशीलता में कई विशेषताएं हैं जैसे डाउनवर्ड रेटिंग गति, वृद्धावस्था प्रभाव, मूल्यांकन के समय पर निर्भरता और अवधि प्रभाव। हम इन विशेषताओं को पकड़ने और साहित्य में मॉडलों की सीमाओं को दूर करने के लिए एक गैर-सजातीय मार्कोव पुनर्जागरण प्रक्रिया आधारित क्रेडिट रेटिंग मॉडल का प्रस्ताव देते हैं।

2007-08 के उप-प्रधान संकट, जिसके कारण लेहमन ब्रदर्स जैसे कुछ प्रमुख वित्तीय संस्थानों के पतन के कारण वित्तीय बाजार में संक्रमित जोखिम के महत्व पर जोर दिया गया है। संक्रमित जोखिम की उम्मीद है क्योंकि कंपनियां अलगाव में नहीं चलती हैं और इसलिए आम आर्थिक कारकों या प्रत्यक्ष व्यापारिक लिंक के कारण फर्मों में निर्भरता मौजूद है। इसलिए, इस थीसिस का उद्देश्य एक बहुआयामी ढांचे में एक संक्रमण प्रक्रिया का प्रस्ताव देना है, जो संक्रमित प्रभावों को पकड़ने में सक्षम है। उसी भावना में, हम मैक्रो-इकोनोमी के गतिशील व्यवहार को शामिल करने के लिए एक संक्रमण-स्विचिंग ढांचे में संक्रमण प्रक्रिया का विस्तार करते हैं, इस तथ्य से प्रेरित है कि वित्तीय प्रतिभूतियों की कीमतें अर्थव्यवस्था की स्थिति द्वारा महत्वपूर्ण रूप से प्रभावित होती हैं, जो स्वयं ही अधीन होती है विभिन्न शासनों के लिए। इसके अलावा, यह थीसिस इन मॉडलों का उपयोग क्रेडिट पोर्टफोलियो डेरिवेटिव के मूल्य निर्धारण के लिए करता है। एक ही विषय के साथ जारी रखते हुए, हम फर्म मूल्यों के लिए प्रॉक्सी के रूप में कार्य करने वाली संपत्ति की कीमतों में कूद क्लस्टरिंग की घटना को पकड़ने के लिए एक हॉक्स कूद-प्रसार संरचनात्मक क्रेडिट जोखिम मॉडल का प्रस्ताव देते हैं।

अंत में, यह थीसिस मार्कोव चेन क्रेडिट रेटिंग मॉडल का प्रस्ताव देने का प्रयास करती है जिसमें फर्म के स्वास्थ्य को नियंत्रित करने वाले संरचनात्मक चर में मौजूद जानकारी शामिल है। प्रस्तावित मॉडल एक हाइब्रिड मॉडल है जो इस संरचना में एक संरचनात्मक मॉडल दोनों का लाभ लेने के लिए डिज़ाइन किया गया है कि यह डिफ़ॉल्ट रूप से यथार्थवादी वर्णन देता है।

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