A THESIS ON

A FUNCTIONAL-ANALYSIS APPROACH TO THE OPTIMAL CONTROL OF DISTRIBUTED-PARAMETER SYSTEMS

BY

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C E R T I F I C A T E

This is to certify that the thesis entitled, ' A Functional-Analysis Approach to the Optimal Control of Distributed-Parameter Systems', which is being submitted by Mr. Ram Nandan Prasad Singh for the award of Degree of DOCTOR OF PHILOSOPHY (Electrical Engineering) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

This thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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A C K N O W L E D G E M E N T S

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S Y N O P S I S

Optimal-control problems of a class of linear distributed-parameter systems with an inequality constraint on the control function can be approached by the methods of Distributed Maximum Principles (Butkovskii, Sage), Dynamic Programming of Bellman or Krein's L-Theory of Moments. However, the optimal-control problems with more than one inequality constraint on the control or state variables, known as the multiple-constraint optimal-control problems, have not been given much attention in the past. Such problems frequently arise in Engineering Practice, especially in metallurgical and chemical industries. In this thesis, a technique of Functional-Analysis approach has been applied to solve the multiple-constraint optimal-control problems for a class of linear distributed-parameter systems which are strictly infinite dimensional ones. A linear distributed-parameter system with a generalised quadratic-performance index and with multiple constraints on the control function has been investigated in Hilbert spaces. The method of Krein's L-Theory of moments has been extended to the time-optimal control of linear distributed systems with multiple constraints on the control and state variables. A controllability criterion for linear distributed systems using the moment method has been presented. The function-theoretic approach yields a unified method of solutions to the above problems

in both lumped and distributed systems.

A brief description of the contents of the chapters included in the thesis is given below:

CHAPTER-I

This includes a general description and background of the problem. Physical motivations and mathematical difficulties are clearly stated.

CHAPTER-II

Historical reviews of references dealing with lumped and distributed systems have been discussed with special reference to the problem under consideration. Various techniques including the functional-analysis approach have been presented. Analysis shows that a distributed system gives rise to a non-harmonic (generalized) Fourier series.

CHAPTER-III

This deals with the minimal-time control of a class of linear distributed systems. In this chapter, two types of minimal-time problems have been attempted:

i) problems with a composite-norm constraint, and
ii) problems with a multiple-norm constraint on the control function.

The method of L-Theory of moments has been applied to obtain the solutions of problems mentioned above. The first type of problems uses the inequality constraint on a generalized quadratic functional and its solution requires the solution of the following two auxiliary problems:

i) An infinite set of Fredholm integral equations

ii) A minimization problem in the dual space.

The second type of problems uses a multiple-norm constraint on the control function. This multiple-constraint is reduced to an equivalent composite-norm functional. The solution gives rise to three auxiliary problems, namely,

i) An infinite set of integral equations in L_1 - space

ii) A functional- minimization in the L_1 - space

iii) A minimization problem in the dual space.

CHAPTER-IV

The optimal control problem of a class of linear distributed-parameter systems with a generalized quadratic performance index has been analysed in Hilbert spaces. Three cases of optimal controls are considered:

i) Control without any constraint,

ii) Control with a general norm-constraint,

iii) Control with a multiple-constraint.

A necessary and sufficient condition for the optimality has been obtained over a weekly compact set in a Hilbert space using the solution of the unconstrained problem.

CHAPTER-V

This chapter deals with the controllability of linear systems using the moment theory method. A problem of Controllability of physical systems leads to the study of a general interpolation problem. The analysis is based on the application of a bi-orthonormal set of functions (comprised of the system's characteristic functions and its conjugates). A criterion for total state controllability of a class of linear distributed systems has been presented in both Hilbert and Banach spaces. The theory has been illustrated by an example.

CHAPTER-VI

In this chapter the results of the problems solved above are discussed. The possible extensions and generalizations of the problems are pointed out for future work.

The thesis is based on the following nine papers:

1) "Functional Analysis Approach to Optimal Control Problems with Multiple Constraints on the Controlling Function", International Journal of Control, Vol.9 No.1 pp.89-101 (January,1969).

2) " Time-optimal Control of a Linear Distributed-Parameter System", Proc. of the Symposium on Control Theory, May,1968, I.I.Sc., Bangalore-12, pp.389-402.

Also published in the Journal of the Institution of Engineers (India), Electronics and Telecommunication Division, Vol.49 No.9, Pt.ET-3,(May 1969).

3) "A Unified Approach to a State-space Model for Linear Distributed Systems"; International Journal of Control (to appear).

4) " Minimal-time Control of Linear Distributed Systems with a Composite-norm Constraint", International J. of Control (to appear).

5) " On the Controllability of Infinite-Dimensional Linear Systems", I.E.E.E.; Trans. on Automatic Control (to appear).

6) "Time-optimal Control of Distributed Systems with a Multiple-Constraint on the Control Function" SIAM, J. on Control (communicated).

7) " Optimal Control of Distributed-Parameter Systems in Hilbert spaces", SIAM J. Control (Communicated).

8) " On the Controllability of Infinite Dimensional Systems in Banach Spaces", SIAM, J. Control (Communicated)

9) " Multiple Constraints in Optimal Control Problems", Proc.I.E.E.E. (Communicated).

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