

**ON SOME MULTISTEP METHODS FOR THE NUMERICAL INTEGRATION
OF STIFF ORDINARY DIFFERENTIAL EQUATIONS**

By

C. Prabhakara Rao

**This is submitted to the Indian Institute of Technology,
Delhi for the award of the Degree of**

DOCTOR OF PHILOSOPHY

**Department of Mathematics
Indian Institute of Technology, Delhi**

December 1974

C E R T I F I C A T E

This is to certify that the thesis entitled "On some multistep methods for the numerical integration of stiff ordinary differential equations" which is being submitted by Mr. C. Prabhakara Rao for the award of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

S. R. K. Iyengar

(S. R. K. Iyengar)
Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-110029

ACKNOWLEDGEMENTS

I am extremely grateful to Dr. S.R.K. Iyengar, Assistant Professor, Department of Mathematics, I.I.T., Delhi, for his valuable guidance, and kind help throughout my research work.

I express my deep sense of gratitude to Prof. M.K. Jain, Professor of Mathematics and Dean of Administration, I.I.T., Delhi, with whom I had many useful discussions, for the keen interest he had evinced in the progress of my work.

I sincerely thank Prof. K.R. Parthasarathy, Head of the Department of Mathematics, I.I.T., Delhi, and Prof. M.P. Singh, Professor of Mathematics, I.I.T., Delhi for their encouragement.

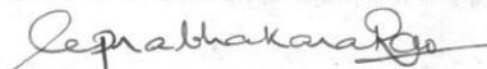
My profound thanks are due to Prof. S.K. Lakshmana Rao, Professor of Mathematics, Regional Engineering College Warangal, for his inspiration and encouragement.

I thank the authorities of the Regional Engineering College, Warangal, for deputing me to this Institute. My thanks are due to the Ministry of Education, Government of India for providing me financial assistance under the Quality Improvement Programme.

I thank the authorities of the Indian Institute of Technology for providing me a fellowship for a part of my stay here, and for making available all research facilities to me.

My thanks are also due to the staff of the Computer Center, I.I.T., Delhi, who have helped me in the computational work.

Finally, I thank Mr. D.R. Joshi for his commendable typing.


(C. Prabhakara Rao)

CONTENTS

| Chapter | | Page |
|---------|---|----------|
| | SYNOPSIS | |
| I | INTRODUCTION | |
| | 1.1 Initial value problems for first order ordinary differential equations | 1 |
| | 1.2 General linear multistep methods | 3 |
| | 1.3 Numerical stability of a multistep method | 8 |
| | 1.4 Hybrid methods | 15 |
| | 1.5 The problem of stiffness and stiffly stable methods | 17 |
| | 1.6 Description of the thesis Figure | 24 |
| II | HIGH ORDER NUMERICAL DIFFERENTIATION TYPE FORMULAS WITH AN OFF-STEP POINT FOR STIFF ORDINARY DIFFERENTIAL EQUATIONS | |
| | 2.1 Introduction | 29 |
| | 2.2 Derivation of the formulas | 31 |
| | 2.3 Derivation of the truncation error | 36 |
| | 2.4 Stability of the formulas | 41 |
| | 2.5 Stiff stability of the formulas | 43 |
| | 2.6 Predictors at k and θ Tables and Figures | 49 53 |
| III | NUMERICAL DIFFERENTIATION TYPE FORMULAS OF HIGHER ORDER WITH AN ARBITRARY NUMBER OF OFF-STEP POINTS | |
| | 3.1 Introduction | 63 |
| | 3.2 Derivation of the formulas | 65 |
| | 3.3 Stability of the formulas | 72 |
| | 3.4 Predictors at k and θ_1 | 73 |

| | | |
|-----------|--|-----|
| | 3.5 Numerical example | 74 |
| | 3.6 Conclusions | 76 |
| | Tables | 78 |
| IV | NUMERICAL DIFFERENTIATION FORMULAS WITH VARIABLE STEP SIZE | |
| | 4.1 Introduction | 90 |
| | 4.2 Non uniform α -mesh | 92 |
| | 4.3 Generalized numerical differentiation formulas (GDF) | 93 |
| | 4.4 Stability of the formulas | 98 |
| | 4.5 Stiff stability properties of the formulas | 100 |
| | Appendix | 102 |
| | Table and Figures | 106 |
| V | ON OPTIMAL STIFFLY STABLE METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS | |
| | 5.1 Introduction | 108 |
| | 5.2 Optimal approximation and optimal difference formulae | 110 |
| | 5.3 Example | 114 |
| | Table | 117 |
| | REFERENCES | 118 |