SOME CONTRIBUTION TO NÖRLUND AND RELATED SUMMABILITY

by

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CERTIFICATE

This is to certify that the thesis entitled 'Some Contribution to Nörlund and Related Summability' which is being submitted by Mr. Ashok Kumar for the award of Degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafied research work. He has worked for the last four years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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SYNOPSIS

This thesis incorporating some aspects of Nörlund and related summability includes four chapters. In the first chapter, a problem concerning the generalised Fekete methods has been considered. In the next two chapters, strong functional (i.e. function-to-function) Nörlund summability has been defined and some of its properties investigated. The last chapter deals with the extension of M. Riesz's mean value theorem for infinite integrals.

In [26], Schaefer has extended the methods introduced by Fekete [11] by replacing the Taylor method with a more general regular summability method 'A', which he calls "A-Nörlund" and "Nörlund-A" and denotes them by $F(A,p_n)$ and $G(A,p_n)$ respectively. He has shown that the transformations $F(A,p_n)$ and $G(A,p_n)$ are the same where A is either the Taylor method or the Euler method. The question arises: What methods A(satisfying suitable restrictions) are such that the $F(A,p_n)$ and $G(A,p_n)$ transformations are identical for bounded sequences and for all positive regular Nörlund methods (N,p_n) ? The aim of the author, in the first chapter, is to find the necessary and sufficient conditions in the direction indicated above. A few known and unknown special cases have also been considered.

The concept of strong Nörlund summability $[N,p_n]_\lambda, \lambda > 0$, of series (and sequences) was introduced and investigated by Borwein and Cass [2,3]. In the second chapter, a definition of strong functional Nörlund summability has been given and some relations between strong functional and functional Nörlund summability have been obtained. It is also shown that if one functional Nörlund method includes the other then the same is true of the associated strong methods.

Recently, Kuttner and Thorpe [19] have established relations between the convergence fields of the strong Nörlund summability methods $[N,q_n]_1$ and $[N,p_n]_1$, where $q(z) = p(z)\ r(z)$ and where $r(z)$ satisfies appropriate restrictions. In the third chapter, analogous results involving strong functional Nörlund summability have been obtained.

Extensive research literature exists in which studies have been made to extend the Riesz's mean value theorem for finite and infinite integrals. More important from the author's point of view as incorporated in chapter four are the contributions of Isaacs [16] and Bosanquet [4]. Isaacs has extended the mean value theorem for infinite integrals and Bosanquet has obtained the results for finite integrals by replacing the functions $t^{\alpha-1}/\Gamma(\alpha)$ and $t^{-\alpha}/\Gamma(1-\alpha)$ by general functions $G(t)$ and $H(t)$ such that

$$\int_0^y G(y-t)\, H(t)dt = 1 \quad \text{for } y > 0, \; = 0 \quad \text{for } y = 0. \qquad \text{(a)}$$

In the fourth chapter, an extension of Isaacs' result has been obtained for general functions $G(t)$ and $H(t)$ satisfying relation (a). The cases of equality have also been discussed.

TABLE OF CONTENTS