

STUDIES ON OPTIMAL PLACEMENT OF MULTIPLE INERTIAL SENSORS FOR IMPROVING SENSING ACCURACY

Nitesh Sahu



Centre for Applied Research in Electronics

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STUDIES ON OPTIMAL PLACEMENT OF MULTIPLE INERTIAL SENSORS FOR IMPROVING SENSING ACCURACY

by

Nitesh Sahu

Centre for Applied Research in Electronics

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Certificate

This is to certify that the thesis entitled “**Studies on optimal placement of multiple inertial sensors for improving sensing accuracy**” being submitted by **Mr. Nitesh Sahu** to the Centre for Applied Research in Electronics (CARE), Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is the record of the bonafide research work carried out by him under our supervision. In our opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.



(Prof. Arun Kumar)

CARE

Indian Institute of Technology Delhi

Hauz Khas, New Delhi 110016

India



(Prof. Rajendar Bahl)

CARE

Indian Institute of Technology Delhi

Hauz Khas, New Delhi 110016

India

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Abstract

Inertial Navigation Systems (INSs) are crucial self-contained systems utilized in military, aviation, and aerospace applications to provide uninterrupted Position, Velocity, and Attitude (PVA) information. These systems rely on Inertial Measurement Units (IMUs), typically comprising three orthogonal accelerometers and gyroscopes, to ensure reliable and robust performance. To address sensor malfunctions and enhance system reliability, Redundant Inertial Measurement Units (RIMUs) have been widely adopted, offering improved sensing accuracy and uninterrupted operation. Fault Detection and Isolation (FDI) in redundant sensor systems plays a vital role in achieving optimal performance. Additionally, sensor configuration significantly impacts the sensing accuracy and hence the navigation performance, and various methods have been explored to determine optimal configurations.

This Ph.D. thesis addresses key challenges in optimizing sensor configurations for redundant inertial sensors. Firstly, we propose a novel algorithm based on the MM (Majorization-Minimization or Minorization-Maximization) principle to determine the optimal orientation of multiple single-axis inertial sensors, considering varying accuracies. Existing research has mostly considered sensors with equal accuracies and uncorrelated noise. Numerical simulations validate the algorithm's effectiveness, demonstrating improved sensing accuracy for practical applications.

The presence of correlated measurement noise in inertial sensors introduces additional complexities in achieving optimal sensor configurations. Correlated noise arises when noise in one sensor is related to noise in another sensor due to common mode noise or shared environmental factors. To address this issue, we propose iterative algorithms based on the MM principle to solve optimization problems with a general noise

covariance matrix. These algorithms offer an effective means to determine optimal configurations when correlated measurement noise is present.

To enhance sensing accuracy, a common approach involves adding additional sensors to existing configurations. We present an approach that preserves the initial sensor configuration while determining optimal orientations for appended sensors. This approach is particularly useful when redesigning the entire configuration is impractical or undesirable. Our method accommodates sequential appending and considers sensors of different accuracies and correlated noise, providing a generalized approach for appending sensors.

Lastly, the thesis addresses the importance of sensor allocation and orientation in achieving an optimal configuration for RIMUs. Poor sensor allocation can lead to the huge lever arm effect (caused by deviations between the center of the RIMU and individual sensors' centers), causing errors in linear acceleration estimation and compromising system accuracy. We formulate the problem of optimal sensor allocation as an optimization problem and propose a numerical approach to determine the optimal allocation of sensors or lever arm vectors. Our approach considers sensors of varying sizes and addresses symmetric, asymmetric, and random configurations, contributing to the development of accurate and reliable navigation systems.

In conclusion, this thesis addresses critical aspects of optimizing sensor configurations for redundant inertial sensors. It introduces novel algorithms, addresses optimal configuration for varying accuracies and correlated measurement noise, explores appending sensors, and emphasizes the significance of sensor allocation. The proposed algorithms and methods offer valuable insights into achieving optimal sensing accuracy and thus maximizing the navigation performance. The findings contribute to advancements in inertial navigation technology and pave the way for the development of more accurate and efficient navigation systems.

सार

इनरशियल नेविगेशन सिस्टम्स (INSs) महत्वपूर्ण आत्म-निहित सिस्टम होते हैं जो सैन्य, विमानन, और अंतरिक्ष अनुप्रयोगों में निर्बाध पोजीशन, वेलोसिटी, और एटिट्यूड (PVA) जानकारी प्रदान करने के लिए उपयोग किए जाते हैं। ये सिस्टम इनरशियल मेजरमेंट यूनिट्स (IMUs) पर निर्भर होते हैं, जिनमें आमतौर पर तीन ऑर्थोगोनल एक्सेलेरोमीटर और गाइरोस्कोप शामिल होते हैं, ताकि विश्वसनीय और मजबूत प्रदर्शन सुनिश्चित किया जा सके। सेंसर की खराबी को संबोधित करने और सिस्टम की विश्वसनीयता बढ़ाने के लिए, रेडंडेंट इनरशियल मेजरमेंट यूनिट्स (RIMUs) को व्यापक रूप से अपनाया गया है, जिससे बेहतर सेंसिंग सटीकता और निर्बाध संचालन प्राप्त होता है। फॉल्ट डिटेक्शन और आइसोलेशन (FDI) रेडंडेंट सेंसर सिस्टम्स में अनुकूल प्रदर्शन प्राप्त करने में महत्वपूर्ण भूमिका निभाता है। इसके अलावा, सेंसर का कॉन्फिगरेशन सेंसिंग सटीकता और इसलिए नेविगेशन प्रदर्शन को महत्वपूर्ण रूप से प्रभावित करता है, और अनुकूल कॉन्फिगरेशन निर्धारित करने के लिए विभिन्न विधियों का पता लगाया गया है।

यह पीएच.डी. थीसिस रेडंडेंट इंटरशियल सेंसरों के लिए सेंसर कॉन्फिगरेशन्स को अनुकूलित करने में महत्वपूर्ण चुनौतियों का समाधान करती है। सबसे पहले, हम एक नया एल्गोरिथ्म प्रस्तावित करते हैं जो MM (मेजराइजेशन-मिनिमाइजेशन या माइनोराइजेशन-मैक्सिमाइजेशन) सिद्धांत पर आधारित है, जो विभिन्न सटीकताओं को ध्यान में रखते हुए कई सिंगल-एक्सिस इंटरशियल सेंसरों की इष्टतम ओरिएंटेशन का निर्धारण करता है। मौजूदा शोध ने ज्यादातर समान सटीकताओं और असंबद्ध शोर वाले सेंसरों पर विचार किया है। संख्यात्मक सिमुलेशनों से एल्गोरिथ्म की प्रभावशीलता की पुष्टि होती है, जो व्यावहारिक अनुप्रयोगों के लिए सेंसरिंग सटीकता में सुधार को दर्शाता है।

इंटरशियल सेंसरों में संबंधित मापन शोर की उपस्थिति इष्टतम सेंसर कॉन्फिगरेशन्स प्राप्त करने में अतिरिक्त जटिलताएं पैदा करती है। संबंधित शोर तब उत्पन्न होता है जब एक सेंसर में शोर एक अन्य सेंसर में शोर से संबंधित होता है, जो सामान्य मोड शोर या साझा पर्यावरणीय कारकों के कारण होता है। इस मुद्दे को हल करने के लिए, हम सामान्य शोर कोवैरिएंस मैट्रिक्स के साथ अनुकूलन समस्याओं को हल करने के लिए MM सिद्धांत पर आधारित पुनरावर्ती एल्गोरिदम प्रस्तावित करते हैं। ये एल्गोरिथ्म संबंधित मापन शोर की उपस्थिति में इष्टतम कॉन्फिगरेशन्स निर्धारित करने के लिए एक प्रभावी साधन प्रदान करते हैं।

सेंसरिंग सटीकता बढ़ाने के लिए, एक सामान्य दृष्टिकोण मौजूदा कॉन्फ़िगरेशन्स में अतिरिक्त सेंसर जोड़ना शामिल है। हम एक दृष्टिकोण प्रस्तुत करते हैं जो प्रारंभिक सेंसर कॉन्फ़िगरेशन को संरक्षित रखते हुए नए सेंसरों के इष्टतम ओरिएंटेशन का निर्धारण करता है। यह दृष्टिकोण विशेष रूप से उपयोगी होता है जब संपूर्ण कॉन्फ़िगरेशन को फिर से डिज़ाइन करना अव्यवहारिक या अवांछनीय होता है। हमारी विधि क्रमिक जोड़ने की सुविधा प्रदान करती है और विभिन्न सटीकताओं और संबंधित शोर वाले सेंसरों पर विचार करती है, जिससे सेंसरों को जोड़ने के लिए एक सामान्यीकृत दृष्टिकोण मिलता है।

अंत में, यह थीसिस RIMUs के लिए इष्टतम कॉन्फ़िगरेशन प्राप्त करने में सेंसर आवंटन और ओरिएंटेशन के महत्व को संबोधित करती है। खराब सेंसर आवंटन के कारण लेवर आर्म इफ़ेक्ट (जो RIMU के केंद्र और व्यक्तिगत सेंसरों के केंद्रों के बीच विचलन के कारण उत्पन्न होता है) हो सकता है, जिससे रेखीय त्वरण अनुमानों में त्रुटियाँ और सिस्टम सटीकता में कमी आ सकती है। हम सेंसरों या लेवर आर्म वेक्टरों के इष्टतम आवंटन को निर्धारित करने के लिए एक संख्यात्मक दृष्टिकोण प्रस्तावित करते हैं, जो विभिन्न आकारों के सेंसरों पर विचार करता है और सममित, विषम, और रैंडम कॉन्फ़िगरेशन्स को संबोधित करता है, जिससे सटीक और विश्वसनीय नेविगेशन सिस्टमों के विकास में योगदान मिलता है।

संक्षेप में, यह थीसिस रेडंडेंट इंटरशियल सेंसरों के लिए सेंसर कॉन्फ़िगरेशन्स को अनुकूलित करने के महत्वपूर्ण पहलुओं को संबोधित करती है। यह नए एल्गोरिथ्म प्रस्तुत करती है, विभिन्न सटीकताओं और संबंधित मापन शोर के लिए इष्टतम कॉन्फ़िगरेशन्स को संबोधित करती है, सेंसरों को जोड़ने की विधि की खोज करती है, और सेंसर आवंटन के महत्व पर जोर देती है। प्रस्तावित एल्गोरिथ्म और विधियाँ इष्टतम सेंसिंग सटीकता प्राप्त करने और इस प्रकार नेविगेशन प्रदर्शन को अधिकतम करने में मूल्यवान अंतर्दृष्टि प्रदान करते हैं। ये निष्कर्ष इंटरशियल नेविगेशन प्रौद्योगिकी में प्रगति में योगदान करते हैं और अधिक सटीक और कुशल नेविगेशन सिस्टमों के विकास का मार्ग प्रशस्त करते हैं।

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Abbreviations

INS	Inertial Navigation System
PVA	Position, Velocity, and Attitude
IMU	Inertial Measurement Unit
RIMU	Redundant Inertial Measurement Unit
RINS	Redundant Inertial Navigation System
FDI	Fault Detection and Isolation
MEMS	MicroElectroMechanical Systems
PSD	Power Spectral Density
FOM	Figures Of Merit
GDOP	Geometric Dilution Of Precision
MM	Majorization-Minimization or Minorization-Maximization
EM	Expectation-Maximization
CG	Center of Gravity
DC	Difference of Convex
MSE	Mean Squared Error
SDP	SemiDefinite Programming
CCP	Convex-Concave Procedure

Notations

Boldface lowercase letters	Column vector
Boldface uppercase letters	Matrix
$()^{-1}$	Inverse of a matrix
$()^T$	Transpose
$\det()$	Determinant of a matrix
$\text{Tr}()$	Trace of a matrix
$\log()$	Logarithm to the base e
x_n	n^{th} element of a vector $\mathbf{x}$
$\mathbf{I}_a$	$a \times a$ identity matrix
$\ \cdot\ _2$	l_2 norm
$\mathbf{A} \succeq \mathbf{B}$	Matrix $\mathbf{A} - \mathbf{B}$ is positive semidefinite
$\mathbb{S}_+^n$	Set of $n \times n$ symmetric positive semidefinite matrices
$ \cdot ^2$	Absolute squared value
$\mathbb{R}$	Real field
$\lambda_{\max}(\mathbf{R})$	Maximum eigenvalue of $\mathbf{R}$
$\text{diag}(\mathbf{x})$	Diagonal matrix formed with a vector $\mathbf{x}$ as its diagonal
$\mathbf{A}(i, :)$	i^{th} row of matrix $\mathbf{A}$