SELF FOCUSING OF ELECTROMAGNETIC WAVES IN SEMICONDUCTORS

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PREFACE

The nonlinear electromagnetic behavior of semiconductors and dielectrics has aroused a great deal of interest in physic and engineers in the last decade, owing to its wide applicatio in signal generation and amplification, and guided wave propag tion particularly at microwave and laser frequencies. Althoug a vast amount of literature[1-3] on this subject is available, vigorous research in this field is being currently pursued. The power of a source is usually limited by technical restrictions and therefore, it is becoming growingly important to concentrate the available power to a narrow region. The concentration of power is very useful for nonlinear applicatio because the efficiency of a nonlinear mechanism increases rapidly with the strength of electromagnetic fields.

The guidance of electromagnetic waves at optical frequencies has been successfully achieved by using the specia designed fibres in which dielectric constant is maximum on the axis and decreases monotonically away from it. This inhomogen provides a mechanism for guidance of the beam. Uchida et al.[4] and Kita et al.[5] have verified experimentally the satisfactory performance of such fibres (usually known as 'Selfoc' fibres)

Recently a vast amount of work has also been reported o the self focusing of laser beams in nonlinear media[6-13]. In many nonlinear media, the dielectric constant is an increasin function of wave intensity. When such a medium is subjected to an electromagnetic beam with nonuniform intensity distribu along its wavefront, the medium behaves as optically inhomoge ous. In most of the commonly excited modes, wave intensity i maximum on the beam axis and decreases away from it, hence the induced inhomogeneities tend to focus the beam. The beam is simultaneously influenced by the diffraction phenomenon. The basic physics of self focusing of an intense electromagnetic beam can be understood by considering the propagation of a uniform parallel cylindrical beam, passing through a medium, in which the variation of refractive index with the field is given by

$$n = n_0 + n_2 E E^* \quad , \tag{I}$$

where n_0 is the field independent part of the refractive index, $n_2 E E^*$ is its field dependent part and n_2 is some coefficient, determined from the nature of nonlinearity. Eq.(I) is valid only at relatively small intensities such that

$$n_2 E E^* << n_0 \quad .$$

If the amplitude of the electric vector is A_0 and th absorption of the beam by the material is neglected, the refractive index in the space illuminated by the beam is $n_0 + n$ while that outside it is n_0 . It should of course be rememb that in fact the intensity decreases continuously with r , (t distance of a point from the axis of the beam) the presently assumed discontinuity in intensity at the boundary of a cylinder, is a convenient approximation, justified by more rigorous analysis. Hence rays making an angle θ with the ax will suffer total internal reflection when

$$\theta < \theta_c \quad , \tag{II}$$

where θ_c ($\frac{\pi}{2} - \theta_c$ being the critical angle) is given by

$$\cos\theta_c = \sin\left(\frac{\pi}{2} - \theta_c\right) = \frac{n_0}{n_0 + n_2 A_0^2} \quad .$$

Since both and are , one gets

$$1-\left(\theta_c^2/2\right) = 1-\left(n_2 A_0^2/n_0\right)$$

or $$\theta_c^2 = 2 n_2 A_0^2/n_0 \quad . \qquad (\text{III}$$

A beam of radius a may be considered as obtained from the incidence of a plane wave front on an aperture of radius a . We know that on account of Fraunhoffer's diffraction all the power of the beam will not be carried by rays parallel to the axis but there will be a directional distribution of intensity. A very large fraction of the power will be carried by rays making an angle less than θ_d with th axis, where θ_d is given by

$$\theta_d = 0.61\lambda_0/2a n_0 \quad ,$$

where λ_0/n_0 is the wavelength of the radiation in the mater and the first minimum of the diffraction pattern is along $\theta =$.

The condition that the beam does not lose power appreciably is equivalent to rays making an angle less than with the axis, being not lost from the beam; this is only possible when

$$\theta_c > \theta_d \quad . \qquad (\text{I}$$

so that such rays will suffer total internal reflection at t boundary and hence return to the beam.

Thus the condition for the focusing of the beam is

$$\theta_c > \theta_d \quad ;$$

which reduces to

$$P > P_{cr} ,$$

where $P = \pi a^2 n_0 \frac{c}{4\pi} A_0^2$ is the power of the beam and P_{cr} is the critical power of the beam given by

$$P_{cr} = \frac{1.22^2 \lambda_0^2 c}{256 n_2} . \qquad (\text{VII})$$

It is thus seen that when $P < P_{cr}$, $\theta_c < \theta_d$ and hence the beam will diverge. For $P = P_{cr}$, $\theta_c = \theta_d$ and hence the beam will propagate uniformly without convergence and divergence We may further extrapolate, as may be shown from rigorous analysis also (cf. Akhmanov et al.[7]), that when $P > P_{cr}$, $\theta_c > \theta$ the convergence of the beam will occur. On defining two important parameters, viz., R_d (the diffraction length) and R_{nl} (the self focusing length),

$$R_d \simeq \frac{a}{\theta_d} \simeq \frac{ka^2}{2} \qquad (\text{VIII})$$

and

$$R_{nl} = a\left(\frac{n_0}{2 n_2 A_0^2}\right)^{1/2} \qquad (\text{IX}$$

The conditions for convergence and divergence are likewi $R_d > R_{nl}$ and $R_d < R_{nl}$ respectively; $k = 2\pi n_0/\lambda_0$ and λ is the free space wavelength. Thus, one can consider the nonlinear medium to act as a converging lens of focal length F while the phenomenon of diffraction can be approximated by the behavior of a diverging lens of focal length R_d .

The above theory is in reasonable agreement with experiments in dielectrics (e.g. CS_2).

It is very instructive to realize the avalanche nature the self focusing effect. Consider a beam propagating in a nonlinear medium, defined by Eq.(I). As the beam gets focused due to nonlinearity, the intensity increases causing enhancement of the nonlinearity and hence in the extent of nonlinear focusing.

The studies on self focusing of electromagnetic waves reported so far are confined to dielectrics. We have made the first attempt to study the self focusing of electromagnetic waves in semiconductors on account of free carrier nonlinearities. In case of semiconductors the nonlinearity arises due to the nonlinear dependence of the free carrier current density $\underline{J}$ on the electric vector $\underline{E}$. For a wave of frequency ω the effective dielectric constant ϵ_{eff} of a semiconductor can be defined as

$$\epsilon_{eff} = \epsilon_L - \frac{4\pi i \sigma}{\omega} , \qquad (X)$$

$$\sigma = \sigma_r - i\,\sigma_i ; \qquad (Xa)$$

where σ is the conductivity and ϵ_L is the lattice contribution to the dielectric constant. Hence

$$\epsilon_{eff} = \epsilon_r - i\,\epsilon_i , \qquad (XI)$$

where

$$\epsilon_r = \epsilon_L - \frac{4\pi \sigma_i}{\omega}$$

and

$$\epsilon_i = \frac{4\pi \sigma_r}{\omega}$$

For low intensities of electromagnetic beam σ_r and σ_i are constant which implies that the current density J is proportional to the electric vector E . For higher intensities of the wave σ_r and σ_i depend on the magnitude of the electric vector and consequently ϵ_r and ϵ_i are functions of the electric vector. For not very high powers of the beam ($P \sim 10$ watts, $E \sim 1$ esu at microwave frequencies) this relationship is quadratic in nature i.e.

$$\epsilon_{eff} = \epsilon_{r_0} - i\epsilon_{i_0} + \epsilon_{r_2} EE^* - i\epsilon_{i_2} EE^* \quad (XII)$$

In the present thesis we have carried out study of this nonlinearity in semiconductors under various situations of interest.

It is instructive to recapatulate the physical processes, leading to the dependence of the conductivity of a semiconductor on the electric vector.

In equilibrium (i.e. in the absence of an electric field), the temperature of the free carriers is the same as that of the crystal so that the net energy exchange between the carriers and the lattice of the crystal is zero. When an electric field is applied free carriers gain energy from the field, and hence acquire a temperature higher than that of the crystal. In the steady state the power gained by the carriers from the electric field equals the power lost by the carriers to the lattice (by collisions). For moderate values of the electric field the rise in the temperature of the carriers is proportional to the square of the electric field. The change in the temperature of the carriers leads to corresponding changes in the collision frequency and effective mass of carriers; the second effect is important in the case of III-V semiconductors e.g. InSb. The first effect is not important in the focusing of the waves.

Further, if the beam is inhomogeneous i.e. the electric vector does not have the same value along a wavefront, the carrier temperature will also vary along the wave front. In the steady state the net carrier current is zero and hence a carrier temperature distribution also leads to a distribution of carrier concentration along the wave-front. Since the carrier concentration is related to the effective dielectric constant, we obtain in effect a distribution of dielectric constant along the wave front, which in turn leads to self focusing of the beam. This effect is not very appreciable in case of semiconductors with a single type of carriers due to the phenomenon of space charge. However when both types of carriers are present the mechanism discussed in this paragraph becomes very important in the self focusing of the beam.

The free carrier nonlinearities appear at relatively low powers (~ 10 watts) and are therefore, important in the interaction of moderately strong microwaves with semiconductors. It is important to mention here that besides these focusing mechanisms in semiconductors the power of the beam is simultaneously lost due to absorption by the carriers. The non linear effects become less and less effective with the advancement of the wave. Thus to obtain appreciable focusing of microwaves, the focusing length due to nonlinearity should be less than the diffraction length ($\sim \frac{1}{2} k a^2$, k being the propagation vector and a is the dimension of the beam) and also the damping length; the damping length is $L_D \simeq (2\epsilon$

It may also be mentioned here that the free carriers are important only when the carrier concentration is high i.e. when the effective plasma frequency is comparable with the wave frequency.

We have presented an elementary version of the quantitati considerations for evaluating the nonlinear dielectric constant in the cases discussed above. Starting with the expressions for dielectric constant thus obtained the focusing of the waves in the semiconductors has been studied by solving the wave equation in the WKB approximation. The intensity distribution has been taken to be Gaussian at the entrance and paraxial ray approximation has been used to obtain analytical expression for the amplitude and transverse dimension of the beam.

A chapterwise summary of the thesis can be given as follows:-

I. <u>Self-Focusing of Electromagnetic Waves by Nonlocal Nonlinear Effects in Semiconductors</u>

We have suggested a new type of nonlinearity which is prominent when both types of carriers, i.e. holes and electrons are present in the semiconductor. To separate the effect of the dependence of m on T_e we have considered the case of germaniu where m does not depend on T_e. It has been shown that the self focusing of electromagnetic waves due to free carrier nonlinearities is possible in feasible sample lengths for easily obtainable powers of the beam. For the focusing of higher frequency waves, the power of the beam has to be enhanced considerably because ϵ_2 decreases with increasing ω very rapidly. The nonlinearities, important in self focusing appears through the redistribution of carriers; the relaxation time nonlinearity is not significant in causing self focusing. It is also important to mention here that at low lattice temperatures, the rise in random energy of carriers (above the thermal equilibrium value) due to the power absorption from the wave is much greater than the drift energy. Therefore the nonlinearities are mainly due to the former.

II. Self-Focusing of Microwaves in Nonparabolic Semiconductors in Presence of a Magnetic Field

In this chapter a study of the mutual interaction of two propagating modes in nonparabolic semiconductors in presence of a static magnetic field has been made. To reduce the mathematical complexity, the case of n-InSb has been chosen where nonlinearity appears through the energy dependence of carrier mass. The focusing of two modes has been found to be much different. As a result a very large change in the ellipticity of the wave along the direction of propagation is obtained. The nonlinear effects in the ellipticity predominate over the linear effects. In the helicon limit, focusing length of the order of one centimeter is predicted whenever axial electric field exceeds 0.3 esu. With increasing carrier concentration the focusing effect is, however, reduced. The focusing in this limit is important when $(\omega - \omega_c) > \nu$ and $\omega/\omega_c >$ These phenomena are useful in having a good insight into the nonlinear mechanisms in the sample because in some instances polarization and phase measurements are easier than the amplitude measurements.

III. Damping Criterion for the Focusing of Lasers in Semiconductors

For the focusing of high frequency (optical range) waves in a feasible length, the sample requires high carrier concentrations. To achieve this, the crystal has to be doped with impurities. This addition of impurities gives rise to enhanced scattering of electrons and hence the absorption of the wave increases. The criteria for the appreciable focusing to occur are that

$$R_{nl} < k_i^{-1} \text{ and } R_{nl} < R_d ,$$

that is the attenuation and diffraction divergence should be less important than the self focusing effect. With the increasing doping in the sample, the damping length (k_i^{-1}) decreases and much of the power of the wave is lost in a short distance. Thus the absorption opposes the focusing. When the damping is very rapid, the focusing effect is almost negligible. A quantitative analysis of the problem has been given by us in this chapter. The limit on maximum frequency that can be focused in the sample has been obtained in the perturbation approximation. For example in n-InSb the maximum frequency that can be focused at 77°K is 10^{15} c/s.

IV. Self-Focusing of Step Pulse Microwaves in Nonparabolic Semiconductors

In this chapter we have studied the nonsteady focusing of electromagnetic waves in n-type InSb, where the nonlinearity arises due to the energy dependence of the effective mass. The energy relaxation time τ_0 of carriers in InSb at low temperat is of the order of 10^{-8} sec. and hence the relaxation time for nonlinearity is also of the same order. When a strong pulsed beam having a duration ($t \sim 10^{-8}$ sec) comparable with the energy relaxation time propagates in the semiconductor, initially ($t << \tau_e$) no focusing occurs because the nonlinearity is set up only in a time of the order of τ_e ; therefore the beam suffers diffraction divergence and the axial intensity decreases. For the latter portion of the pulse ($t \gtrsim \tau_e$) the nonlinearity is set up and hence the beam gets focused; the axial intensity is therefore enhanced. Thus the pulse is distorted very much and the distortion may be used as a direct method to study the energy relaxation process inside the sample. We have carried out an adequate analysis of the problem taking into account the relaxation effects. For a typical case of n-InSb, when $E_o \sim .1$ esu the effective focusing length for the tail portion of the pulse comes out to be $\sim$ .5 cm when the sample is at 77°K.

V. Self-Distortion of an Amplitude Modulated Signal in a Self Focusing Nonlinear Medium

In the case of amplitude modulated continuous waves, the focusing distorts the amplitude envelope considerably. An amplitude modulated wave may be expressed as

$$E = E_o (1 + \mu \cos \Omega t) e^{i(\omega t - kz)} , \qquad (XIII)$$

where μ is the index of modulation, Ω is the modulation frequency and ω is the carrier frequency. The distortion of the signal inside the semiconductor can be understood like this.

Let us take the modulation period $T_m = \frac{2\pi}{\Omega}$ to be much greater than the energy relaxation time. Then the nonlinearity in the sample would be more when the amplitude of the wave is more. Hence the high amplitude portion of the wave is focused more than the portion of minimum amplitude.

We have carried out the quantitative analysis of wave distortion in n-InSb in the case when the dispersion of side bands can be neglected i.e. when the velocities of side bands are taken to be equal to that of the carrier wave. This type of distortion i.e. over modulation of the wave is basically different from the usual nonlinear demodulation which appears due to nonlinear absorption. In case of a beam with average power $\sim$ 1 watt, the index of modulation has been found to increase by four times at $\Omega = 10^8$ sec^{-1}.

VI. Change of Polarization of a Focusing Beam and the Effects of Sample Size

The problem of polarization rotation of Gaussian beams based on the WKB solution of wave equation in self focusing fibres has been investigated in this chapter. It has been found that near the focusing region the change of polarization is appreciable. Finite longitudinal size of the fibres reduces the effective length of the samples for wave focusing.

The work reported in the thesis has resulted in the following publications by the author:-

1. Nonlinear Mechanisms for self-focusing of Microwaves in Semiconductors, J.Appl.Phys., In Press 1973.
2. Damping Criterion for Focusing of Laser Beams in Semiconductors, J.Phys.D:Appl.Phys., In Press 1973.

3. Effects of Self-focusing on the self distortion of amplitude Modulated Microwaves in Nonparabolic Semiconductors, Opto-Electronics (UK), In Press 1973.
4. Microwave Faraday Rotation and Self focusing of Helicon Waves in n-InSb, J.Appl.Phys., August 1973.
5. Analysis of Propagation in a 'Selfoc' Fibre: The Axial Field Component and the Effect of Multiple Reflections, Opto-Electronics (UK), 4, 51 (1972).

In addition to the work reported in detail in the thesis, the author has also to his credit the following publications in allied fields:

1. Resolution of Point Sources by a Dust-Laden Telescopic Objective, Optik, 6, 569 (1970).
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