

A Thesis
on

DIFFERENCE METHODS FOR THE SOLUTION OF SOME
PARABOLIC AND HYPERBOLIC PARTIAL DIFFERENTIAL EQUATIONS

by

RAMESH CHAND MITTAL

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Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-110029

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