

**PONDEROMOTIVE EFFECTS IN NONLINEAR  
WAVE-PLASMA INTERACTION AND  
ROLE OF CHAOS**

By

**UMA RAMANATHAN**

A thesis submitted in partial fulfilment  
of the requirements for the degree of  
**DOCTOR OF PHILOSOPHY**



**FUSION STUDIES PROGRAM  
Center of Energy Studies  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
NEW DELHI-110016 INDIA**

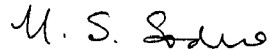
March 1988


TO MY PARENTS

CERTIFICATE

This is to certify that the thesis entitled "PONDEROMOTIVE EFFECTS IN NONLINEAR WAVE-PLASMA INTERACTION AND ROLE OF CHAOS" being submitted by Ms. UMA RAMANATHAN to the Fusion Studies Program, Centre of Energy Studies, Indian Institute of Technology, Delhi for the award of the degree of 'Doctor of Philosophy' is a record of the bonafide research work carried out by her. Ms. Uma Ramanathan worked under our guidance for the submission of this thesis which to our knowledge has reached the requisite standard.

This thesis or any part thereof has not been submitted to any other university/institution for the award of any degree or diploma.

M. S. Sodha

Dr. D. Subbarao

Professor M.S. Sodha\* and Dr. D. Subbarao\*\*

(Supervisors)

Fusion Studies Program

\*Department of Physics and \*\*Centre of Energy Studies

Indian Institute of Technology, Delhi

New Delhi - 110016

## ABSTRACT

The physics of nonlinear wave-plasma interaction is central to the understanding of the plasma processes related to heating, confinement and stability of fusion plasmas. This thesis deals with the three essential aspects of this problem : the nonlinear coupling force between the wave and the plasma (ponderomotive force), the resultant selfconsistent nonlinear wave fields and the particle dynamics in these selfconsistent fields including chaotic dynamics all of which are interrelated. Particularly the emphasis is on the role of plasma selfconsistency effects on the onset of chaos in the particle dynamics. The latter limits the validity of the ponderomotive theory on which the study of wave plasma interaction is based.

The investigation of this thesis is in three parts each of which is devoted to each of the aspects mentioned above. Two wave-plasma model problems are studied : (1) when the ponderomotive force is spatially periodic and (2) when the frequency of the wave resonates with the characteristic plasma frequency. Novel results in finite amplitude wave propagation are obtained. The phenomenon of nonlinear Bragg reflection is established as a scattering process in competition with parametric instabilities like SBS and owes its existence to space periodic ponderomotive force which arise in a large class of problems particularly relevant to fusion (chap.-3). Also ponderomotive effects near cyclotron resonance give rise to bistability and jump phenomena in plasma dielectric and magnetisation property (chap.-2).

The selfconsistent particle dynamics in nonlinearly standing electrostatic waves is shown to exhibit chaos at thresholds lower compared to that in nonselfconsistent fields; the ponderomotive effect combined with kinetic effects such as nonlinear frequency shift due to the trapping of particles is seen to be crucial. The concept of ponderomotive force at relativistic powers may break down (chap.-4). Near electron cyclotron resonance, the nonlinear gyrofrequency shift due to selfgenerated magnetic fields is seen to induce period doubling route to chaos in the motion of the electron. In both these cases collective effects manifest through a frequency shift (chap.4).

The study of these two problems leads one to quite general conclusions : the mathematical singularity at plasma resonance can be taken care of by collective effects such as the ponderomotive effects. Period doubling chaos could be generic to plasma resonances and establishes KAM surface break up due to collective effects while for nonresonant regime, hamiltonian chaos is the manifestation of collective effects. In such regimes, the ponderomotive theory needs to be reexamined. Ponderomotive effects then resemble diffusion and a new formalism for the plasma response is required possibly in terms of a discrete and continuous spectrum of Case-Van Kampen-like modes. Such an expansion is particularly good when the motion contains both regular and chaotic parts. The formalism of nonlinear Bragg reflection also opens up a new way of looking at selfconsistent field profiles and densities for saturated parametric instabilities and offers a way of determining fields at resonance.

## PREFACE

Controlled thermonuclear fusion research has entered about the fortieth year with the hope that breakeven may possibly occur in a couple of years. It is well known that the hot plasma formed in this context interacts with the background magnetic and other electromagnetic fields showing up a number of instabilities most of which have interesting nonlinear origins. High power laser interaction with the corona of a laser-produced plasma is an example of a situation which involves nonlinear interactions and is the basis for the concept of the laser-driven-implosion inertial confinement scheme for achieving thermonuclear fusion. Other situations of interest which involve nonlinear electromagnetic wave interactions with plasmas are in radio wave confinement, heating and current drive with background or self consistent magnetic field configurations in the context of magnetic confinement approaches to thermonuclear fusion.

It is the aim of <sup>the</sup> present thesis to study and understand analytically some of the nonlinear phenomena arising in such situations. In view of the fact that at the high temperatures involved in fusion plasmas, electron ion collisions are scarce, a situation of collisionless plasmas is mostly dealt with. The nonlinearities arising out of the so-called ponderomotive force are, therefore, of interest and are dealt with in the beginning. The selfconsistent structures of the plasma and electromagnetic wave in the nonlinear regime are next in importance are, therefore, dealt with in that order ending in the generic concept of nonlinear Bragg reflection. The recently popularised fact

that such self-consistent concepts of integrability of the nonlinear problems is not always possible is given much consideration and as such the effects of chaos are dealt in the last part of the thesis.

Much evolution in the techniques of handling nonlinear interactions has taken place elsewhere or brought to light during the evolutions of the present thesis. A reflection of these trends will be found in the thesis partly accounting for the bulk of the thesis which could not do away with some pedagogy. At the same time, possibly much more could have been achieved particularly more numerical or simulational support for the calculations on aspects of chaos and are left along with problems mentioned at the end of the thesis for the future.

A more comprehensive discussion of nonlinear processes in electromagnetic wave <sup>interaction</sup> intecation with plasmas which places the thesis in context is given in the introductory first chapter. A flair for applications of the phenomena discussed in the thesis is also given there. Specific applications are discussed during the course of the development of the thesis. The emphasis, however, is on methodology and basic phenomena throughout.

The thesis would not have been possible without the support of many. A word of thanks is in order.

## ACKNOWLEDGEMENT

I would like to express my deep gratitude to Dr.D.Subbarao who , as my supervisor , taught me to expect order out of chaos in research.I am particularly thankful to him for introducing me to the exciting field of chaos which has given this thesis a completely new turn.A good amount of the work in this thesis evolved from stimulating discussions with him and I am much grateful for his constant guidance and patience.

I would like to express my sincere thanks to Prof.Sodha who as my advisor and head of the Plasma-Fusion group provided encouragement and a peaceful environment to work in.

I would also like to thank Prof.Ghatak for his encouragement; collaborating with him on self-focusing has been of much use to me.

I also sincerely thank Prof.V.K.Tripathi for his comments and criticism on my work.Thanks are due to Dr.L.A.Patel who guided my work during the first one and a half years of work.Though I changed the course of my research after he left the institute , the initial exposure to various topics in plasma physics has been beneficial.

Finally I acknowledge my deep gratitude to my parents, my sister and my grandparents who always gave moral support by way of "high expectations" and continuously encouraged my academic endeavour.

Thanks are due to Mr.Anil Khandelwal for taking care of the difficult job of typing the thesis.

The NCERT for its National Science Talent Search Scholarship which financially supported for most of the period spent on this thesis, the CSIR Senior Research Fellowship which supported the final year of research and the Royal Commission for the Exhibition of 1851, London which is supporting the last months of completion of this thesis along with the hospitality of the Dynamics Group at Queen Mary College ,London and particularly the warm encouragement of Prof.Ian Percival are all happily acknowledged.

*RUma*

( UMA RAMANATHAN )

## CONTENTS

	Page
Abstract	
Preface and Acknowledgement	
<u>Chapter 1</u> : INTRODUCTION	1
<u>Chapter 2</u> : PONDEROMOTIVE FORCE OF ELECTRO- MAGNETIC WAVES	24
2.1 Introduction	26
2.2 Ponderomotive Force of homogeneous plane Electromagnetic waves in a magnetoplasma	57
2.3 <span style="border: 1px solid black; padding: 2px;">Resonant Ponderomotive Force and         its effects</span> <span style="border: 1px solid black; padding: 2px;">EFFECTS OF SELFINDUCED MAGNETISATION         NEAR CYCLOTRON RESONANCE</span>	69
2.4 Conclusion	89
<del>Abstract</del> : References	89
<u>Chapter 3</u> : SPACE PERIODIC PONDEROMOTIVE FORCE AND NONLINEAR ELECTROMAGNETIC WAVE PROPAGATION	92
3.1 Introduction	95
3.2 Nonlinear Bragg Reflection from Ummagnetised Plasmas	102
3.3 Nonlinear Wave Propagation of Arbitrary Polarisation in Magnetoplasma	155
3.4 Discussion	191

	Appendix	203
	References	208
<u>Chapter 4</u> :	CHAOS IN NONLINEAR WAVE PLASMA INTERACTION	211
4.1.	Introduction to Chaos	214
4.2	Chaos in Standing Waves- Unmagnetised Plasma	295
<del>4.3</del>	<del>Appendices</del>	<del>364</del>
4.3	Chaos in Electrostatic Waves in a Magnetised Plasma	369
4.4	Diffusion	392
4.5	Dissipative Chaos Near Cyclotron Resonance	411
4.6	Conclusion	419
	References	422
<u>Chapter 5</u> :	SUMMARY AND PROBLEMS FOR THE FUTURE	435
	List of Publications	439
	Reprints of some of the publications	441