

PROCESSING METHODS FOR LEFT-RIGHT AMBIGUITY RESOLUTION IN TWINLINE TOWED ARRAY SONAR

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Processing Methods for Left-Right Ambiguity Resolution in Twinline Towed Array Sonar

by

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Dedicated to

My Family

Certificate

This is to certify that the thesis entitled “**Processing Methods for Left-Right Ambiguity Resolution in Twinline Towed Array Sonar**” being submitted by **Mr. Baiju M Nair** to the Centre for Applied Research in Electronics, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is the record of the bona-fide research work carried out by him under our supervision. In our opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.

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Abstract

A linear array with omnidirectional sensors cannot discriminate between the azimuth angle and its complementary angle due to axisymmetry about the array axis. This thesis examines solutions to the bearing ambiguity problem known as Left-Right (LR) ambiguity using two closely spaced parallel arrays. In the first part of the thesis, we design a Second-order Nulling (SoN) beamformer to produce a cardioid-type spatial response with a null that is electronically steered to the complementary angle of the direction of interest. Three methods, namely SoN, Null Constrained Beamformer (NCB), and 2-Dimension Direct Beamformer (2DDB), are studied. The performance is evaluated in correlated noise conditions modeled by a first-order auto-regressive process. This work establishes analytical results for the bound on the detection gain of the two parallel arrays in terms of the Rayleigh quotient of the circulant noise correlation matrix.

A closed-form expression is developed for estimating the loss due to cardioid processing at small inter-array spacing. The closed-form expressions are validated using Monte-Carlo simulations based on the deflection criterion. Twin array processing using the 2DDB method results in a higher Detection Gain (DG), while cardioid-based beamformers give a higher Rejection Ratio (RR). Data from an at-sea experiment using a towed array is analyzed. Field results show that the deeper null achieved by the cardioid method results in a higher RR compared to 2DDB. From the field data, approximately 19 dB Left-Right discrimination is demonstrated at 1650 Hz for the broadside. Experimental results from a towed array corroborate the analysis and simulations.

In the second problem addressed in the thesis, we examine the role of a sparse

array, specifically the Coprime Sparse Arrays (CSA) on the twin array. CSA with two colocated Uniform Linear Array (ULA) are used to estimate $\mathcal{O}(MN)$ direction of arrivals for a narrow band source with only $\mathcal{O}(M + N)$ sensors. However, CSA cannot discriminate the azimuth and its complementary angle due to the symmetry along the array axis. We design LR resolved CSA (LRCSA) to produce a null that is electronically steered to the complementary angle of the desired direction. LRCSA twin array achieves higher spatial resolution than the conventional fully populated LR array with the same number of sensors, much like the colocated CSA. The bound on the DG is analytically established for the LRCSA under correlated noise scenarios. Twin array processing using the LRCSA method results in narrow main lobe beamwidth along with a higher rejection ratio (RR) compared to 2DDB and NCB. Data obtained from a sea experiment using a towed array is analyzed. The results confirm that CSA based cardioid processing resolves LR ambiguity with a higher RR and narrower beams.

The performance from an adaptive beamformer-based LR ambiguity resolution with twin-array is examined next. We use an adaptive beamformer based on Minimum Variance Distortionless Response (MVDR) beamformer along with cardioid beamforming to produce a null that is electronically steered to the complementary angle of the desired direction for a twin array. In the adaptive scheme, beamforming is first performed along the azimuth to obtain two sets of beams. The beam signal from each array is then combined linearly with weights determined to minimize the output power at a complementary angle while maintaining a unity response in the desired direction. The minimization principle tends to develop a null in the complementary direction, while the unity response constraint ensures an undistorted signal from the desired direction. We demonstrate the performance of MVDR based beamformer on a twin array in the correlated noise modeled by a first-order auto-regressive process.

This thesis also explores a Deep Neural Network (DNN) technique for LR resolution and analyzes its performance. We develop a framework for the left-right resolution problem based on the probability of error in estimating the correct state (PoE). Binary classification based on DNN is used to resolve the LR ambiguity. The DNN method based on raw sensor data processing is found to be closer to the optimal performance.

अमूर्त

सर्वदिशात्मक सेंसर वाला एक रैखिक सरणी सरणी अक्ष के बारे में अक्ष समरूपता के कारण अजीमुथ कोण और उसके पूरक कोण के बीच भेदभाव नहीं कर सकता है। यह थीसिस दो निकट दूरी वाले समानांतर सरणियों का उपयोग करके बाएं-दाएं (एलआर) अस्पष्टता के रूप में ज्ञात असर अस्पष्टता समस्या के समाधान की जांच करती है। थीसिस के पहले भाग में, हम एक शून्य के साथ कार्डियोइड-प्रकार की स्थानिक प्रतिक्रिया उत्पन्न करने के लिए एक दूसरे क्रम के न्यूलिंग (SoN) बीमफॉर्मर को डिज़ाइन करते हैं जो इलेक्ट्रॉनिक रूप से ब्याज की दिशा के पूरक कोण पर संचालित होता है। तीन तरीकों, अर्थात् एसओएन, नल कंस्ट्रेंड बीमफॉर्मर (एनसीबी), और 2-डायमेंशन डायरेक्ट बीमफॉर्मर (2डीडीबी) का अध्ययन किया जाता है। प्रदर्शन का मूल्यांकन प्रथम-क्रम ऑटो-रिग्रेसिव प्रक्रिया द्वारा निर्मित सहसंबद्ध शोर स्थितियों में किया जाता है। यह कार्य सर्कुलेट शोर सहसंबंध मैट्रिक्स के रेले भागफल के संदर्भ में दो समानांतर सरणियों के पता लगाने के लाभ पर सीमा के लिए विश्लेषणात्मक परिणाम स्थापित करता है।

छोटी अंतर-सरणी रिक्ति पर कार्डियोइड प्रसंस्करण के कारण होने वाले नुकसान का अनुमान लगाने के लिए एक बंद-रूप अभिव्यक्ति विकसित की गई है। विक्षेपण मानदंड के आधार पर मॉन्टे-कार्लो सिमुलेशन का उपयोग करके बंद-रूप अभिव्यक्तियों को मान्य किया जाता है। 2डीडीबी विधि का उपयोग करके ट्विन ऐरे प्रसंस्करण के परिणामस्वरूप उच्च डिटेक्शन गेन (डीजी) होता है, जबकि कार्डियोइड-आधारित बीमफॉर्मर उच्च अस्वीकृति अनुपात (आरआर) देते हैं। खींचे गए सरणी का उपयोग करके समुद्र में किए गए प्रयोग के डेटा का विश्लेषण किया जाता है। फ़ील्ड परिणाम बताते हैं कि कार्डियोइड विधि द्वारा प्राप्त गहरे शून्य के परिणामस्वरूप 20dB की तुलना में उच्च आरआर होता है। फ़ील्ड डेटा से, ब्रॉडसाइड के लिए 1650 हर्ट्ज़ पर लगभग 19 डीबी बाएँ-दाएँ भेदभाव प्रदर्शित होता है। खींचे गए सरणी से प्रायोगिक परिणाम विश्लेषण और सिमुलेशन की पुष्टि करते हैं।

थीसिस में संबोधित दूसरी समस्या में, हम एक विरल सरणी की भूमिका की जांच करते हैं, विशेष रूप से जुड़वां सरणी पर कोप्राइम विरल सारणी (सीएसए)। केवल $O(M + N)$ सेंसर वाले एक संकीर्ण बैंड स्रोत के लिए आगमन की $O(MN)$ दिशा का अनुमान लगाने के लिए दो कोलोकेटेड यूनिफ़ॉर्म लीनियर ऐरे (ULA) वाले CSA का उपयोग किया जाता है। हालाँकि, सीएसए सरणी अक्ष के साथ समरूपता के कारण अजीमुथ और उसके पूरक कोण में भेदभाव नहीं कर सकता है। हम एलआर रिजॉल्व्ड सीएसए (एलआरसीएसए) को एक शून्य उत्पन्न करने के लिए डिज़ाइन करते हैं जो इलेक्ट्रॉनिक रूप से वांछित दिशा के पूरक कोण पर संचालित होता है। एलआरसीएसए ट्विन ऐरे समान संख्या में सेंसर के साथ पारंपरिक पूरी तरह से आबादी वाले एलआर ऐरे की तुलना में उच्च स्थानिक रिज़ॉल्यूशन प्राप्त करता है, जो कि कोलोकेटेड सीएसए की तरह है। सहसंबद्ध शोर परिदृश्यों के तहत एलआरसीएसए के लिए डीजी पर सीमा

विश्लेषणात्मक रूप से स्थापित की गई है। एलआरसीएसए विधि का उपयोग करके जुड़वां सरणी प्रसंस्करण के परिणामस्वरूप 2डीडीबी और एनसीबी की तुलना में उच्च अस्वीकृति अनुपात (आरआर) के साथ संकीर्ण मुख्य लोब बीमविड्थ होता है। एक खींचे गए सरणी का उपयोग करके समुद्री प्रयोग से प्राप्त डेटा का विश्लेषण किया जाता है। परिणाम पुष्टि करते हैं कि सीएसए आधारित कार्डियोइड प्रसंस्करण उच्च आरआर और संकीर्ण बीम के साथ एलआर अस्पष्टता को हल करता है।

ट्विन-एरे के साथ एक अनुकूली बीमफॉर्मर-आधारित एलआर अस्पष्टता रिज़ॉल्यूशन से प्रदर्शन की जांच आगे की जाती है। हम कार्डियोइड बीमफॉर्मिंग के साथ-साथ मिनिमम वेरिएंस डिस्टॉरशनलेस रिस्पॉन्स (एमवीडीआर) बीमफॉर्मर पर आधारित एक अनुकूली बीमफॉर्मर का उपयोग करते हैं, जो एक नल का उत्पादन करता है जिसे इलेक्ट्रॉनिक रूप से एक जुड़वां सरणी के लिए वांछित दिशा के पूरक कोण पर ले जाया जाता है। अनुकूली योजना में, बीम के दो सेट प्राप्त करने के लिए बीमफॉर्मिंग को पहले अज़ीमुथ के साथ किया जाता है। प्रत्येक सरणी से बीम सिग्नल को वांछित दिशा में एकता प्रतिक्रिया बनाए रखते हुए एक पूरक कोण पर आउटपुट पावर को कम करने के लिए निर्धारित वजन के साथ रैखिक रूप से जोड़ा जाता है। न्यूनतमकरण सिद्धांत पूरक दिशा में शून्य विकसित करता है, जबकि एकता प्रतिक्रिया बाधा वांछित दिशा से एक अविरल संकेत सुनिश्चित करती है। हम प्रथम-क्रम ऑटो-रिग्रेसिव प्रक्रिया द्वारा सहसंबंधित शोर में एक जुड़वां सरणी पर एमवीडीआर आधारित बीमफॉर्मर के प्रदर्शन को प्रदर्शित करते हैं।

यह थीसिस एलआर रिज़ॉल्यूशन के लिए डीप न्यूरल नेटवर्क (डीएनएन) तकनीक की भी खोज करती है और इसके प्रदर्शन का विश्लेषण करती है। हम सही स्थिति (पीओई) का अनुमान लगाने में त्रुटि की संभावना के आधार पर बाएं-दाएं समाधान समस्या के लिए एक रूपरेखा विकसित करते हैं। डीएनएन पर आधारित बाइनरी वर्गीकरण का उपयोग एलआर अस्पष्टता को हल करने के लिए किया जाता है। कच्चे सेंसर डेटा प्रोसेसिंग पर आधारित DNN विधि इष्टतम प्रदर्शन के करीब पाई गई है।

Glossary

Boldfaced lower case symbols denote vectors and boldfaced upper case symbols denote matrices respectively. $\mathbf{x}^H$ denotes Hermitian (conjugate-transpose) of $\mathbf{x}$.

$\mathbf{a}_{ULA}$	Array response vector for the linear array.
$\mathbf{A}_{ULA}$	Array response matrix for the linear array.
$\mathbf{A}_T$	Array response matrix for the twin array.
$\mathbf{a}_T$	Array response vector for the twin array.
g_T	Twin array beamformed signal only output.
$\mathbf{n}_b$	Noise received at the b array.
$\mathbf{n}_c$	Noise received at the c array.
n_T	Twin array beamformed noise only output.
$\mathbf{r}$	Range.
$\mathbf{R}_N$	Spatial noise correlation between sensors in an array.
$\mathbf{R}_{N,ij}$	Spatial noise correlation between i^{th} and j^{th} sensors across single linear array.
$\mathbf{R}_\mathcal{N}$	Spatial noise correlation between sensors across twin array.
$\mathbf{R}_{S,ij}$	Spatial signal correlation between any i^{th} and j^{th} sensors across single linear array.
$\mathbf{R}_\mathcal{S}$	Spatial signal correlation between sensors across twin array.
Σ_N	Diagonal Value.
$\mathbf{w}_{ULA}$	Weight vector for linear beamforming.
$\mathbf{w}_T$	Weight vector for twin beamforming.
$\mathbf{w}_{T,2D}$	Weight vector for twin beamforming using 2-D beamforming.
$\mathbf{w}_{T,NCB}$	Weight vector for twin beamforming using Null Constrained beamforming.
$\mathbf{w}_{T,SoN}$	Weight vector for twin beamforming using Second Order Beamforming.
$\mathbf{x}$	Signal received at the array.

$\tilde{\mathbf{x}}_b$	Signal received at the b array.
$\mathbf{x}_c$	Signal received at the c array.
y_b	Beamformed output at the b array.
y_c	Beamformed output at the c array.
$\tilde{y}_b$	Sparse array beamformed output at the b array.
$\tilde{y}_c$	Sparse array beamformed output at the c array.

Acronyms

2DDB	2-Dimension Direct Beamformer.
3DDB	3-Dimension Direct Beamformer.
3NCB	Three Sensor Null Constrained Beamformer.
ASW	Anti-submarine Warfare.
AUV	Autonomous Underwater Vehicle.
CBF	Conventional Beamforming.
CLRCSA	Correlated CSA based processing for twin array.
CSA	Coprime Sparse Arrays.
CSA_{min}	CSA based on minimum processing technique.
CSA_{pp}	CSA based on product processing technique.
CSDM	Cross-spectral density matrix.
DG	Detection Gain.
DNN	Deep Neural Network.
DOA	Direction of Arrival.
HPBW	Half-Power Beamwidth.
LR	Left-Right.
LRCSA	LR resolved CSA.
$LRCSA_{min}$	LRCSA based on minimum processing technique for twin array.
$LRCSA_{pp}$	LRCSA based on product processing technique for twin array.
$LRULA_{2D}$	LR processing with ULA using 2D CBF.
$LRULA_{NCB}$	LR processing with ULA using Null Constrained beamformer.
$LRULA_{SoN}$	LR processing with ULA using Second order Nulling beamformer.
MVDR	Minimum Variance Distortionless Response.

NCB	Null Constrained Beamformer.
PoE	Probability of error in estimating the correct state.
PSL	Peak Side Lobes.
RIS	Real and Imaginary component of the sensor signal.
RR	Rejection Ratio.
SNR	Signal to Noise Ratio.
SoN	Second-order Nulling.
ULA	Uniform Linear Array.

Contents

Certificate	i
Acknowledgements	ii
Abstract	iv
Glossary	vi
Acronyms	viii
List of Figures	xiii
List of Tables	xx
1 Introduction	1
1.1 Beamforming	2
1.2 Towed Array Sonar (TAS)	3
1.3 Challenges/ Problems Identified	5
1.4 Contributions	11
1.5 Organization of the Thesis	13
2 Sensors, Signal-Noise Model and the Towed Array Experiment	16
2.1 Introduction	16
2.2 Sensors for Acoustic Measurements	18
2.3 Signal and Noise Models	25
2.4 DG of Uniform Linear Array	26

2.5	Experimental Setup	27
2.6	Summary	29
3	LR Ambiguity Resolution Methods for Twin Arrays	30
3.1	Introduction	30
3.2	Array Signal Model	33
3.3	Left-Right Processing	35
3.3.1	Cardioid Beamforming	36
3.3.2	Direct Beamforming	40
3.4	Performance Evaluation	40
3.4.1	Rejection Ratio (RR)	42
3.4.2	Detection Gain (DG)	45
3.5	Triplet Array Performance	50
3.6	Results of Sea Experiments	54
3.7	Summary	59
4	LR Ambiguity Resolution using Sparse Arrays	60
4.1	Introduction	60
4.2	Array Signal Model	62
4.3	LR Coprime Sensor Array Processing	65
4.4	Performance Evaluation	67
4.4.1	Detection Gain (DG)	67
4.4.2	Rejection Ratio (RR)	74
4.5	Simulation Results and Discussion	75
4.6	Results of Sea Experiments	78
4.7	Summary	83
5	LR Ambiguity Resolution using Adaptive Beamformer	85
5.1	Introduction	85
5.2	Adaptive Beamforming	86
5.3	Analytical Framework	87

5.4	Simulations and Performance Evaluation	90
5.4.1	Performance of Single ULA	90
5.4.2	DG and RR Performance of Twin Array using Adaptive Beam- former	92
5.5	Summary	99
6	LR Ambiguity Resolution as Binary Classification Problem	100
6.1	Introduction	100
6.2	LR Ambiguity Resolution Measure Based on Probability of Error . . .	101
6.2.1	General Case	101
6.2.2	Correlated Noise	105
6.2.3	Simulations and Results	106
6.3	Classification Problem	112
6.3.1	DNN	113
6.3.2	LR using DNN	114
6.4	DNN Simulations	115
6.5	Summary	117
7	Conclusion	119
7.1	Scope for Future Work	120
	Bibliography	122
	Publications based on this Dissertation	138
	Technical Biography of Author	139

List of Figures

1.1	Deployment configuration of the TAS	3
1.2	Illustration of LR ambiguity from beampattern of linear array steered to 60° . The black line indicates the towed array axis.	6
1.3	3D spatial beampattern obtained by beamforming a ULA to steering angle $\phi = 90^\circ$	7
1.4	2D spatial beampattern obtained by beamforming a ULA to steering angle $\phi = 90^\circ$	7
1.5	3D spatial beampattern obtained by beamforming a ULA to steering angle $\phi = 45^\circ$	8
1.6	2D spatial beampattern obtained by beamforming a ULA to steering angle $\phi = 45^\circ$	8
1.7	ULA (Blue circles) compared to a CSA with the same number of sensors $L = 12$. Red squares indicate sensors for the subarray undersampled by $M = 2$ and green triangles for the subarray undersampled by $N = 3$. The extended CSA concatenates the “basic” geometry (dashed box), shown here for $P = 3$ periods. For the same number of sensors, the CSA has greater aperture than the ULA.	9
1.8	Beampattern for linear array steered to 60° with conventional and coprime processing. Sensors of array A and array B are spaced at $D > M\lambda$, with $M > 1$. mulAB is the beampattern based on coprime processing.	11
2.1	Twin Hydrophones case	19

2.2	Spatial patterns obtained using (2.10) with two sensors and different d values	20
2.3	Uniform Linear Array case	21
2.4	Configuration of the twin array	22
2.5	Configuration of the triplet array	24
2.6	View of triplet inside the towed array tube	24
2.7	Configuration of the N-array	25
2.8	Two-Ship Experiment	28
2.9	Track of the two ships: Source ship(Tx) and Rx ship movements. Red line is the stationary ship transmitting the signal, and the blue line is the Track of the ship moving with the towed array.	29
3.1	Configuration of the LR twin array	33
3.2	Angle selection for SoN method	36
3.3	Schematic of the SoN method	38
3.4	Beampatterns with the three LR ambiguity resolution methods using twin array ($2L$ sensors), $L = 10$	41
3.5	Theoretical RR versus steering angle ϕ (signal-only). Twin ULA with each linear array composed of $L=100$ sensors at $d = \lambda/4$ and $\lambda/30$. . .	42
3.6	RR versus steering angle ϕ (SNR = 32 dB). Twin ULA with each array composed of $L=100$ sensors at $d = \lambda/4$ and $\lambda/30$, $\tilde{r} = 0^\circ$	43
3.7	RR versus steering angle ϕ (SNR = 0 dB). Twin ULA with each array composed of $L=100$ sensors at $d = \lambda/4$ and $\lambda/30$, $\tilde{r} = 0^\circ$	43
3.8	RR versus spatial noise correlation ($\alpha = \beta = \rho$) at steering angle $\phi = 90^\circ, 60^\circ, 30^\circ$. $d = \lambda/30$, $\tilde{r} = 0^\circ$	45
3.9	DG versus steering angle (ϕ). Twin ULA with each linear array composed of $L=100$ sensors. $\alpha = \beta = 0$, $\tilde{r} = 0^\circ$	48
3.10	DG versus inter-array spacing d in terms of λ at two different steering angles. $\alpha = \beta = 0$	48
3.11	DG versus spatial noise correlation ($\alpha = \beta = \rho$). Steering Angle $\phi = 90^\circ$, $d = \lambda/10$ and $\lambda/30$	49

3.12	DG versus roll angle ($\tilde{r}$) at steering angle $\phi = 90^\circ, 60^\circ$ and 30° for twin array and triplet array. Twin and triplet arrays have each linear array composed of $L = 100$ sensors. $\alpha = \beta = 0, d = \lambda/30$	50
3.13	RR versus roll angle ($\tilde{r}$) at steering angle $\phi = 90^\circ, 60^\circ$ and 30° for twin array and triplet array. Twin and triplet arrays have each linear array composed of $L = 100$ sensors. $\alpha = \beta = 0, d = \lambda/30$	51
3.14	DG versus spatial noise correlation ($\alpha = \beta = \rho$) at steering angle $\phi = 90^\circ, 60^\circ$ and 30° for twin array and triplet array. Twin and triplet arrays have each linear array composed of $L = 100$ sensors. $\tilde{r} = 0^\circ$	51
3.15	RR versus spatial noise correlation ($\alpha = \beta = \rho$) at steering angle $\phi = 90^\circ, 60^\circ$ and 30° for twin array and triplet array. Twin and triplet arrays have each linear array composed of $L = 100$ sensors. $\tilde{r} = 0^\circ$	52
3.16	DG versus spatial noise correlation ($\alpha = \beta = \rho$) at steering angle $\phi = 90^\circ, 60^\circ$ and 30° for twin array and triplet array. Twin and triplet arrays have each linear array composed of $L = 100$ sensors. $\tilde{r} = 30^\circ$	52
3.17	RR versus spatial noise correlation ($\alpha = \beta = \rho$) at steering angle $\phi = 90^\circ, 60^\circ$ and 30° for twin array and triplet array. Twin and triplet arrays have each linear array composed of $L = 100$ sensors. $\tilde{r} = 30^\circ$	53
3.18	Left-Right resolved, time vs. bearing output for the SoN beamformer at frequencies 1650 Hz (upper) and 750 Hz (lower), respectively. The source emerges from the end-fire at around 15° , moves toward the broadside, and finally disappears at 165° . The Red line indicates the GPS track.	55
3.19	Experimental performance of the beamformers. Beam power vs. bearing for frequencies 1650 Hz (upper) and 750 Hz (lower), respectively. It may be noted that RR for SoN method and NCB are the same and is hidden below.	56

3.20	Experimental RR vs. bearing performance of the three beamformers presented along with the Monte-Carlo simulation result for SoN. Frequencies 1650 Hz ($\alpha = 0.19$ and $\beta = 0.99$) and 750 Hz ($\alpha = 0.6$ and $\beta = 0.99$), respectively. NCB and SoN have same values across bearing. 2DDB has small RR.	58
4.1	Configuration of the LR twin array. Black boxes comprising b-array and green boxes comprising c-array with L^e sensor locations each spaced at $D = \lambda/2$ as the intra-array spacing and $d \ll D$ as the inter-array spacing. a.) Blue circles on b-array and c-array each represent ULA with $L = 10$ sensors and are used for comparison with CSA. b.) Red crosses represent CSA1 with M sensors spaced at $N\lambda/2$ and extended by a factor P . Black diamonds represent CSA2 with N sensors at $M\lambda/2$ spacing. P is the extension factor from single basic CSA. $M = 5, N = 6, P = 1$ are used in this representation.	63
4.2	Schematic of LR resolved CSA (LRCSA) with product processing. . .	65
4.3	Comparison between twin array beampatterns using 2DDB, SoN, LRCSA1, LRCSA2 and LRCSA for $\phi = 80^\circ$, null at $\phi_c = 280^\circ$. Twin ULA used for 2DDB, SoN has $2L$ ($L = 10$) sensors. LRCSA1 ($\tilde{\mathbf{z}}_1$) has $2MP$ whereas LRCSA2 ($\tilde{\mathbf{z}}_2$) has $2NP$ sensors. LRCSA ($\tilde{\mathbf{z}}_{pp}$) has $2L_{co}$ sensors, where L_{co} is generated from $(M, N, P) = (5, 6, 1)$, $d = \lambda/30$	68
4.4	Comparison between twin array beampatterns using 2DDB, SoN, CSA1, CSA2 and LRCSA for $\phi = 310^\circ$, null at $\phi_c = 50^\circ$. Twin ULA used in 2DDB, SoN has $2L$ ($L = 10$) sensors. LRCSA1 ($\tilde{\mathbf{z}}_1$) has $2MP$ and LRCSA2 ($\tilde{\mathbf{z}}_2$) has $2NP$ sensors. LRCSA ($\tilde{\mathbf{z}}_{pp}$) has $2L_{co}$ ($L_{co} = 10$) sensors, where L_{co} is generated from $(M, N, P) = (5, 6, 1)$, $d = \lambda/30$	69
4.5	Maximum ν for different α values of noise correlation $\mathbf{R}_N$ in ULA, $\tilde{\mathbf{R}}_{N,1}$ and $\tilde{\mathbf{R}}_{N,2}$ for CSA with $(M, N, P = 5, 6, 10)$	73

4.6	Analytical results of DG for ULA(L), CSA _{pp} (L_{co}), 2DDB($2L$), SoN($2L$), NCB($2L$) and LRCSA($2L_{co}$). $L = 100$ and coprime pairs ($M, N, P = 5, 6, 10$) has $L_{co} = 100$ sensors. The array is steered to the broadside. Results are presented for inter-array spacing of $d = \lambda/30$ and $d = \lambda/4$ in the case of a twin array.	73
4.7	DG for LRCSA with undersampled subarray ($M, N, P = 5, 6, 10$) is compared with SoN and 2DDB, $d = \lambda/30, \lambda/4$. (a) DG versus spatial noise correlation $\alpha = \beta = \rho$, $\phi = 90^\circ$, (b) DG versus steering angle ϕ , $\alpha = \beta = 0$	77
4.8	DG versus inter-array spacing d , $\alpha = \beta = 0.5$, $\phi = 90^\circ$	78
4.9	$\hat{R}\hat{R}$ for LRCSA with undersampled subarray ($M, N, P = 5, 6, 10$) is compared with SoN and 2DDB, $d = \lambda/30, \lambda/4$.: (a) $\hat{R}\hat{R}$ versus spatial noise correlation $\alpha = \beta = \rho$, $\phi = 90^\circ$, (b) $\hat{R}\hat{R}$ versus steering angle ϕ , $\alpha = \beta = 0$	79
4.10	$\hat{R}\hat{R}$ versus inter-array spacing d , $\alpha = \beta = 0.5$, $\phi = 90^\circ$	80
4.11	Experimental performance of the three beamformers. Beam Power vs. ϕ at 1650 Hz : (a) 80° (b) 50°	81
4.12	Experimental performance of the three beamformers. Beam Power vs. ϕ at 1300 Hz : (a) 80° (b) 50°	82
5.1	Difference ($\delta = \nu_{E,max} - \nu_{A,max}$) between the eigenvalue $\nu_{A,max}$ using analytical bound and the estimated value $\nu_{E,max}$	93
5.2	$\eta_{\mathbf{L},O}$ and $\eta_{\mathbf{L},C}$ variations along steering angle ϕ for a ULA with array composed of $L=100$ sensors at different ρ	94
5.3	$\eta_{\mathbf{L},O}$ and $\eta_{\mathbf{L},C}$ variations along ρ for a ULA with array composed of $L=100$ sensors at different steering angle ϕ	95
5.4	DG for twin ULA and LRCSA with each linear array composed of $L=100$ sensors as a function of, a.) Steering angle, ϕ . b.) Spatial correlation ρ	97
5.5	RR for twin ULA and LRCSA with each linear array composed of $L=100$ sensors as a function of, a.) Steering angle, ϕ . b.) Spatial correlation ρ	98

6.1	PoE vs SNR for array steered to 90° , $\rho = 0$. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	107
6.2	PoE vs SNR for array steered to 90° , $\rho = 0.5$. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	108
6.3	PoE vs SNR for array steered to 90° , $\rho = 0.8$. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	108
6.4	PoE vs ρ for array steered to 90° , SNR = -20 dB. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	109
6.5	PoE vs ρ for array steered to 90° , SNR = -10 dB. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	109
6.6	PoE vs ρ for array steered to 90° , SNR = 0 dB. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	110
6.7	PoE vs SNR for array steered to 5° , $\rho = 0.8$. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	110
6.8	PoE vs SNR for array steered to 55° , $\rho = 0.8$. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	111
6.9	PoE vs steering angle ϕ at SNR = 0 dB and $\rho = 0.8$. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	111
6.10	PoE vs steering angle ϕ at SNR = 10 dB and $\rho = 0.8$. 2DDB has $2L$ ($L = 100$) sensors. LRCSA and CLRCSA has coprime pair as ($M = 5, N = 6, P = 10$).	112

6.11	Basic conceptual design of binary DNN. The input layer consists of multiple inputs and a single output.	113
6.12	The neural network architecture with 3 hidden fully connected (FC) layers, followed by classification of the state. A total of $4L$ inputs from the twin array is the input to the RIS-DNN.	114
6.13	PoE vs. SNR for twin array steered to 60 deg, $d = \lambda/30$	116
6.14	PoE vs ϕ for twin array at SNR =10 dB, $d = \lambda/30$	116
6.15	PoE vs. SNR for with array perturbation $\mathbf{G}_{gb} = \mathbf{G}_{gc} = -10$ dB for twin array steered to 60 deg, $d = \lambda/30$	118

List of Tables

4.1	Theoretical and experimental HPBW using the LR beamformers. $D = 0.37$ m (0.5λ of 2 kHz). 2DDB has $2L$ ($L = 10$) sensors. The LRCSA ($2L_{co}$) array configuration has coprime pair as $(M = 5, N = 6, P = 1)$, $L_{co} = 10$	83
6.1	Accuracy achieved at various SNR using DNN method. The array configuration has $L = 10$ sensors.	117