

**VIBRATION AND DAMPING ANALYSIS OF MULTILAYERED CONICAL SHELLS**

**BY**

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**submitted**

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**to the**

**INDIAN INSTITUTE OF TECHNOLOGY, DELHI**

**OCT 1989**

**TO MY PARENTS**

## CERTIFICATE

This is to certify that the thesis entitled " VIBRATION AND DAMPING ANALYSIS OF MULTILAYERED CONICAL SHELLS" being submitted by Mr Kamal N. Khatri to the Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy in Mechanical Engineering** is a record of bonafide research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for submission of this thesis, which has reached the requisite standard.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Kamal N. Khatri

## ABSTRACT

The governing equations of motion for vibrations of a general multilayered conical shell consisting of an arbitrary number of specially orthotropic material layers have been derived using variational principles. In this analysis, extension, bending and transverse shear of each layer and transverse, rotary and longitudinal translatory inertias of the shell have been considered. A general analysis for determining the vibration and damping characteristics in terms of resonance frequencies and system loss factors for all families of modes of vibration for multilayered conical shell has been developed. Variation of resonance frequencies and associated system loss factors with geometric and material property parameters has been studied for axisymmetric and antisymmetric vibrations of three, five and seven layered constant size conical shells with alternate elastic and viscoelastic layers. Three sets of end conditions have been investigated : simply supported, clamped - clamped and free - free.

The analysis for multilayered conical shell has been extended for vibration and damping analysis of laminated shells of fiber reinforced composite materials. These studies have been carried out by changing cone semi vertex angle of the shell of composite laminates with different stacking arrangements under constant thickness criterion, for various fiber orientations.

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