

**ADAPTIVE FINITE ELEMENT ANALYSIS FOR ACCURATE
COMPUTATION OF STRESSES AND DESIGN
SENSITIVITIES FOR STRUCTURES UNDER
TRANSIENT DYNAMIC LOADS**

By

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Submitted

***in fulfilment of the requirement for
the degree of***

DOCTOR OF PHILOSOPHY



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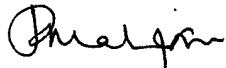
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CERTIFICATE

This is to certify that the thesis entitled 'Adaptive Finite Element Analysis for Accurate Computation of Stresses and Design Sensitivities for Structures under Transient Dynamic Loads' being submitted by Mr. Anjan Dutta to the Indian Institute of Technology, Delhi for the award of the Degree of Philosophy in Applied Mechanics is a record of bonafide research work carried by him under our supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirement for the Doctor of Philosophy Degree.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.



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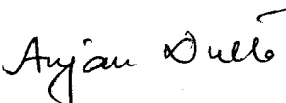
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ABSTRACT

A measure for discretization error is proposed which is a logical extension of the Zienkiewicz-Zhu (1987) error criterion by involving time integration to consider the variation of response with time. Using this error measure, an adaptive mesh refinement strategy is proposed which yields good control over the discretization errors in transient dynamic analysis. An optimal mesh is attained iteratively wherein meshes are refined on the basis of local error indicators so that the discretization errors are within prescribed limit satisfying also the modal cut-off criteria based on the ratios of modal participation factor to its maximum value and modal significance factor (product of modal participation factor and amplification factor) to its maximum value, when the modal superposition method is used. A number of 2-D benchmark problems are solved using this integrated approach where an automatic 2-D quadrilateral mesh capable of designing meshes of varying densities over the domain is utilized. The performance of the integrated module is also tested on plate bending problems using the degenerated shell elements. To demonstrate the usefulness of the algorithm on a practical problem, adaptive analysis has also been utilized for the accurate evaluation of contact force under low velocity impact.

An efficient design derivative computation method for 2-D F.E. problem under transient dynamic loading as proposed by Ramakrishnan et al. (1991) where an adjoint problem is associated with the direct problem is considered for the study of all the numerical errors associated with its computation. Accurate design derivative values with displacement constraints are obtained when an optimal mesh for discretization errors coupled with appropriate number of basis modes satisfying both direct and

adjoint loading conditions is used. Design derivative computation with stress constraints are slightly more complicated because of the complexities of the adjoint loadings. Static correction method needs to be used for proper representation of adjoint load in addition to the strategy adopted for displacement constraints. Studies are also conducted for the design derivative computation using time marching scheme. The method with time marching scheme does not require any adjoint response correction similar to static correction method and yields correct design sensitivities with much coarser time step size; but total computational time is generally higher than that of the modal superposition method. In order to introduce versatility, adaptive time marching scheme is also introduced. Design sensitivities are also computed for plate bending problems using the same algorithm; but programming is more involved because of the complexities of the derivatives.

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