

SUBOPTIMAL CONTROL OF LINEAR SYSTEMS VIA
REDUCED ORDER MODELS

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ABSTRACT

By using the formalism of the linear-quadratic theory, many design problems in control systems are eventually reduced to the solution of a matrix Riccati equation. This is particularly valid for linear regulator and tracking problems. The optimal control is usually implemented in a feedback structure. However, when process industry and Aerospace problems are tackled, the Riccati formalism represents a computationally difficult and cumbersome task, since the order of these systems is large. For an n th-order linear system the number of coupled nonlinear differential equations corresponding to a symmetric matrix Riccati equation is $\frac{1}{2}(n^2+n)$. The available computer installations imposes a limit on the order of the system to be handled. Even if a large computer is available, the convergence problems with large numbers of coupled nonlinear differential equations are serious and may require an excessive amount of computer time. Accordingly, there is a need for obtaining near optimal/sub-optimal solutions for higher order plants. There are different schools of thought for the solution of this problem. One approach is to evaluate the near optimal solutions by perturbation methods which approximate the numerical solution of the Riccati equation. The second method is to evaluate the suboptimal solution by statevectors partitioning and the third approach is to develop a suboptimal control policy via reduced order models. A study of the third approach is presented in this thesis. Design methods derived from this approach are applicable to a large class of linear systems having arbitrary dynamics.

In order to use reduced order models for deriving sub-optimal control strategy, a relationship between the original statevector and the model statevector is required. This relationship is known as aggregation matrix. In this thesis aggregation matrices for reduced order models based on various simplification techniques have been derived. An optimal feedback gain matrix derived from an appropriate lower order model and the corresponding aggregation matrix can then be used to implement a sub-optimal feedback controller for the plant.

Among the various methods available for reducing the order of the systems, the continued fraction technique is simple and requires less computation than all others. However, this technique suffers from the disadvantage of that it may yield an unstable lower order model or a particular order, even when the original higher order system is stable. This problem has been reconsidered and a necessary condition has been derived for selecting a stable lower order model. A further improvement of this technique is also included. In connection with this technique of model simplification, a simple algorithm for continued fraction inversion has also been developed.

In general the stability of the suboptimal control system is not guaranteed. It is shown in this thesis that the resultant feedback system is always stable when it is based on the reduced order model of Davison. Further applications of the reduced order models in pole-allocation problems and for deriving suboptimal controllers with inaccessible states, with derivative constraints and industrial regulators with

constant disturbances via reduced order models is presented. A new approach is investigated for utilizing Chidambara model for deriving a suboptimal control policy. The limits of performance degradation suffered by the suboptimality are developed. It is shown that a considerable saving in computational requirements can be achieved by using reduced order models for linear regulator and pole allocation problems.

A new method of model simplification is developed by matching the frequency response of the original system and the reduced order model.

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