

**MULTIDIMENSIONAL SYSTEMS THEORY :
INVERSION AND REALIZATION**

THEORETICAL STUDIES IN IMAGE PROCESSING AND MODERN CONTROL

BY
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CERTIFICATE

This is to certify that the Thesis entitled,
"Multidimensional Systems Theory: Inversion and Realization",
being submitted by Ajay K.Raina to the Indian Institute of
Technology, Delhi, towards the fulfilment of requirements
for the award of the degree of Doctor of Philosophy in
Electrical Engineering, is a record of bona-fide research
work carried out by him under my supervision and guidance.
This work, to the best of our knowledge, has not been
submitted anywhere else for the award of any degree or
Diploma.



(Dr. T. KARUNAKARAN)

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