

# **FUZZY-ROUGH APPROACH TO PATTERN CLASSIFICATION: HYBRID ALGORITHMS AND OPTIMIZATION**

by

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**Submitted**

**in fulfilment of the requirements of the degree of Doctor of Philosophy**

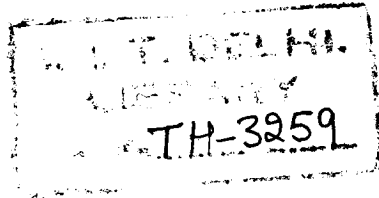
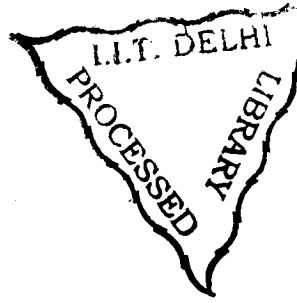
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*With 'Vatsalya' of*  
*My Loving Parents*  
*and*  
*My Younger Brothers*

# CERTIFICATE

This is to certify that the thesis entitled "**Fuzzy-Rough Approach to Pattern Classification: Hybrid Algorithms and Optimization**", which is being submitted by **Mr. Bhatt Rajen Balavantbhai** to the **Department of Electrical Engineering, Indian Institute of Technology Delhi**, for the award of the degree of **Doctor of Philosophy**, is a record of bonafide research work he has carried out under my guidance and, in my opinion, it has reached the standards fulfilling the requirements of the regulations relating to the degree. The results contained in this thesis have not been submitted to any other university or institute for the award of a degree or a diploma.

  
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**ABSTRACT**

The primary objective of any supervised function approximator is to learn an unknown function (or a good approximation of it) from a set of observed input-output patterns. Pattern classification is a special case of function approximation, where each pattern is assigned to a particular class, *i.e.*, the output in classification problem is one of the discrete values corresponding to class rather than real-valued function.

This thesis proposes a fuzzy-rough approach to pattern classification, and develops some hybrid algorithms and optimization techniques for attribute selection and induction of fuzzy decision trees. The major contributions of the thesis are: formulation of hybrid fuzzy-rough measures and their analysis from a pattern classification view point, incorporation of these measures for the development of attribute selection and novel fuzzy-rough decision tree induction algorithms, the development of neural-like parameter adaptation strategies in the framework of neuro-fuzzy decision trees, and the methodology for the structure and initial parameter identification of a generalized class of Gaussian RBF networks based on fuzzy decision trees. The proposed algorithms have been stated explicitly in the formal notation and in *pseudocode* format. Extensive computational experiments have been reported and the proposed algorithms have been experimentally compared with well-known algorithms available in the literature using real-world standard datasets.

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