MINIMAL DEGREE SEQUENCE FOR KNOTS

by

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Department of Mathematics

Submitted

in fulfilment of the requirement of the degree of

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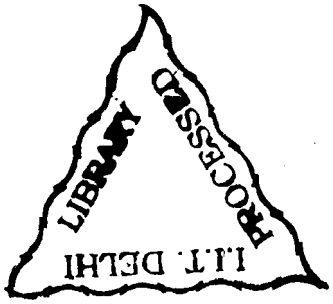

To My Parents

Certificate

This is to certify that the thesis entitled "**Minimal Degree Sequence for Knots**" submitted by **Madeti Prabhakar** to the Department of Mathematics, Indian Institute of Technology Delhi, for the award of the degree of Doctor of Philosophy, is a record of *bona fide* research work carried out by him under my guidance and supervision.

The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The work contained in this thesis has not been submitted to any other university or institute for the award of any degree or diploma.

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Abstract

In this thesis we have addressed the following question: given an open knot-type K, what is the minimal triple $(l, m, n) \in \mathbb{N}^3$ such that there exist real polynomials $f(t), g(t)$, and $h(t)$ of degrees l, m and n respectively and the map $t \mapsto (f(t), g(t), h(t))$ from $\mathbb{R}$ to $\mathbb{R}^3$ represents K? Here minimality is with respect to the usual lexicographic ordering in $\mathbb{N}^3$. We have succeeded in answering this question for all *2–bridge knots* and partially answered it for all *torus knots*. For a general knot-type we have been able to provide only a lower bound for such a triple.

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