NON-LINEAR DYNAMICS AND CHAOS OF FLEXIBLE ROTOR SYSTEMS

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by

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CERTIFICATE

This is to certify that the thesis entitled **'NON-LINEAR DYNAMICS AND CHAOS OF FLEXIBLE ROTOR SYSTEMS'** being submitted by **Tikam chand Gupta** for the award of the degree of Doctor of Philosophy has been prepared under our supervision in conformity with the rules and regulations of the Indian Institute of Technology, Delhi. We further certify that the thesis has attained the requisite standard fulfilling the requirement for the degree of Doctor of Philosophy. The results contained in this thesis have not been submitted, in part or full, to any other University for any degree or diploma.

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ABSTRACT

The present thesis recommends the use of flexible rotor modeling as against rigid rotor modeling, for the purpose of investigation of the effect of ball bearing nonlinearities and varying compliance of bearing on the dynamic response of realistic rotor ball bearing systems. The nonlinearity in the dynamic response is induced by the nonlinear Hertzian deformation force between the balls and races, and the internal radial clearance. The superiority of the flexible rotor modeling as against the rigid rotor modeling has been established successfully by performing detailed dynamic analysis on the following two rotor systems: (i) a flexible horizontal rotor supported on two deep groove ball bearings with radial clearance and (ii) a flexible vertical rotor supported by an angular contact ball bearing mounted at the lower end and a deep groove ball bearing mounted at the upper end. Only theoretical simulation is performed for dynamic response characterization of the horizontal rotor but vertical rotor is studied both theoretically and experimentally. A generalized Timoshenko beam FE formulation is used to model the flexible rotor. The nonlinear dynamic response is quantitatively characterized by various tools like Poincaré maps, Floquet multipliers, time histories, phase plots and Lyapunov exponents.

First, the nonlinear dynamic response of an unbalanced flexible rotor supported on deep groove ball bearing with clearance, is numerically computed using Newmark-β implicit time integration scheme. A significant difference between dynamic responses of rotor at centrally mounted disc location and bearing location is observed, which establishes the need to follow the flexible rotor formulation instead of rigid rotor formulation. Then, a parametric analysis has been carried out to investigate the effect of

different system parameters, such as rotating unbalance, rotational speed, rotor shaft flexibility and bearing radial clearance, on the nonlinear dynamic response of horizontal flexible rotor supported on deep groove ball bearings.

Accounting for the nonlinear dynamic response of a horizontal balanced flexible rotor ball bearing system, a parametric study is theoretically performed to investigate the time-varying nonlinear dynamic stiffness of ball bearing as a function of various time-invariant parameters like flexibility of shaft, radial clearance and, rotational speed.

To capture unstable solutions as well and to further characterize the nonlinear dynamic response for instability, the existing time domain shooting technique has been modified by coupling it with the fixed point algorithm (FPA) and has been used to derive quasi-periodic solutions and Floquet multipliers. The maximum value of Lyapunov exponent is determined using Wolf's algorithm with Gram-Schimdt orgthogonalization. The maximum values of Floquet multipliers and Lyapunov exponents are taken as signatures to determine the instability and chaotic nature of the dynamical system. The instability and chaos in the non-linear dynamic response is found to depend explicitly upon stiffness ratio (shaft stiffness/Hertzian load deflection factor) of rotor ball bearing system, radial clearance of ball bearing, unbalance force and rotational speed. A range of parameters has been identified for which the flexible rotor system is susceptible to instability and/or chaos.

Experiments have been performed on a vertical rotor to capture the effects of self-excitation due to varying compliance of bearing and ball bearing nonlinearities, in the dynamic response. The measured nonlinear dynamic response is analyzed using various analytical tools like orbits, phase plots, displacement-time plots, FFT and Poincaré maps.

A FE model is then formulated to theoretically simulate the dynamic response of the vertical rotor having the same dimensions and specifications as of experimental rig. In an attempt to study instability of the dynamic response of vertical flexible rotor supported on ball bearings, it has been found that the nonlinear dynamic response of the vertical rotor ball bearing system can not be derived successfully by modified shooting method. Therefore, the Newmark-β implicit time integration technique has been used to derive nonlinear dynamic response of vertical rotor.

From the experimental as well as theoretical studies on vertical rotor ball bearing system, it has been found that the orbit of the centrally mounted disc and the rotor centre in upper ball bearing are not concentric with the bearing' axis line and with the increase in rotational speed, the centre of the disc orbit approaches the bearings' axis line. The Poincaré maps and FFT constructed based experimentally measured dynamic response reveal the motion as chaotic. This observation is found to be in contradiction to the conclusion reached by the theoretical analysis.

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