

**NONLINEAR ELASTODYNAMICS OF
CRACKED CONCRETE BEAMS**

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Submitted

in fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

to the



**INDIAN INSTITUTE OF TECHNOLOGY, DELHI
INDIA**

DECEMBER, 2008

Acc. No. TH-3722.....

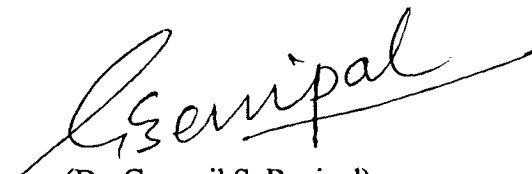


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CERTIFICATE

This is to certify that the thesis entitled, “**Nonlinear Elastodynamics of Cracked Concrete Beams**” being submitted by **Umesh Kumar Pandey** to the Indian Institute of Technology, Delhi for the award of the degree of **Doctor of Philosophy** is a bonafide record of research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirement for the degree of **Doctor of Philosophy**.

The results contained in this thesis have not been submitted, in part or full, to any other University or Institute for the award of any degree or diploma.



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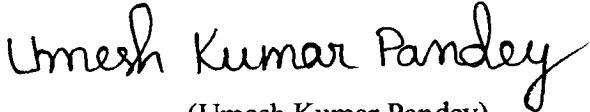
ACKNOWLEDGEMENTS

I express my deep sense of gratitude towards my supervisor Dr. Gurmail S. Benipal for providing inspiration and motivation for the present work at every stage with invaluable suggestions. I have found him ready to help solving my smallest doubts at any moment despite his extremely busy schedule. No amount of appreciation can be good enough to express my gratitude and indebtedness to him.

I am grateful to Prof. K. G. Sharma, Prof Y.Nath, Prof A.K.Jain, Prof. B. Bhattacharjee, Prof. A.K.Nagpal, Prof. T. K. Datta, Prof. Ashok Gupta, Dr. Alok Madan, Dr Suresh Bhalla and Supratic Gupta for their support whenever I was in need.

I take this opportunity to express my thanks to Dr. R. R. Babu, Dr. Umesh Pendharkar, Dr. Sandeep Choudhury and Dr. Amit Agarwal, without their continuous support and moral boosting the journey would have been much difficult. I have a galaxy of friends who have been the source of my continuous strength during the course of this work. I express my thanks towards all my friends who made this arduous task a pleasurable experience indeed.

I would like to acknowledge the blessings of my parents and my elder brother. They have sacrificed a lot for me throughout my life. This thesis is dedicated to them, with thanks for all they have done for me. Finally, I can not forget to mention the continuous support given by my wife and my daughter Ipshita who gave me the liberty of staying away from them for many days at a stretch during the course of this work.


(Umesh Kumar Pandey)

ABSTRACT

During their service life, concrete structures are subjected to various types of dynamic forces. At present, no theory of vibration of cracked concrete structures under working loads is available. The objective of the present Thesis is to study the dynamical behavior of cracked elastic concrete beams.

Cracked concrete beams exhibit different flexural rigidities under sagging and hogging moments. In this Thesis, such SDOF concrete beams have been modeled as bilinear dynamical systems with stiffness and damping coefficients depending upon sense of vibration amplitudes. These beams have been shown to exhibit subharmonic resonances, jump phenomenon and extreme sensitivity to initial conditions, system parameters, sense of sinusoidal force and magnitude of sustained load.

A new class of first order homogeneous dynamical systems valid for Two-DOF concrete beams has been proposed. The displacement space is partitioned into two linear and two nonlinear distinct conical regions. In the nonlinear regions, the stiffness and damping coefficients are functions non-negatively homogeneous of order zero of nodal amplitudes. Depending upon the sustained loads, such beams have been found to execute linear or bilinear small vibrations about equilibrium state. Under large applied sinusoidal forces, these beams exhibit nonlinear dynamical behavior with the system properties changing continuously or abruptly over a large range. Surprisingly, vibrations about the passive state exhibit some aspects of linear behavior alongwith modal subharmonics and combination subharmonics. Effectiveness of nonlinear tuned mass vibration absorbers has also been explored.

The newly proposed bilinear and first order homogeneous oscillators for modeling SDOF and Two-DOF concrete beams respectively constitute significant contributions to

the fields of nonlinear vibrations and of dynamics of concrete beams. There is a paucity of relevant empirical data. The work reported here is expected to be of significant value for design, control and health monitoring of concrete structures. Possible fertile areas for future research have been suggested.

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