

FINITE ELEMENT ANALYSIS OF
PLANE STRESS FRACTURE AND STABLE CRACK GROWTH

RAM NARAIN SINGH

A thesis submitted to the
Indian Institute of Technology, Delhi
for the Award of the Degree of
DOCTOR OF PHILOSOPHY

DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, DELHI

July 1981

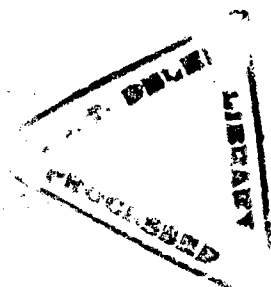
पूज्यामाँ
एवम्
पूज्यपिताजी
के
चरणों
में
सादर
समर्पित

IIT CENTRAL LIBRARY
Acc. No TH-931 De 77/483
Cost: F C IC
Acqd. by S. Call No
Date of Transf. r

TH

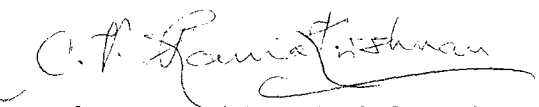
624.013.517.949

SIN - F



CERTIFICATE

This is to certify that the thesis entitled 'FINITE ELEMENT ANALYSIS OF PLANE STRESS FRACTURE AND STABLE CRACK GROWTH', being submitted by Mr. Ram Narain Singh to the Indian Institute of Technology, Delhi, INDIA, for the award of the degree of DOCTOR OF PHILOSOPHY, is a record of bonafide research work carried out by him under our supervision and guidance. The thesis work in our opinion, has reached the standard, fulfilling the requirements for the award of the degree of DOCTOR OF PHILOSOPHY. The research report and the results presented in this thesis have not been submitted, in part or in full, to any other University or Institute, for the award of any degree or diploma.


(Dr. C.V. Ramakrishnan)
Professor
Department of Applied Mechanics
Indian Institute of Technology
New Delhi 110016, INDIA

(Dr. S.N. Sinha)
Assistant Professor
Department of Civil Engineering
Indian Institute of Technology
New Delhi 110016, INDIA

ACKNOWLEDGEMENT

I acknowledge with deep sense of gratitude the help and invaluable guidance by Professor C.V.Ramakrishnan, whose unshakable faith in success provided me with moral support and constant encouragement for carrying out this investigation. The author is extremely grateful to Dr S. N. Sinha for his supervision and constant encouragement during the course of present study. But for their unending enthusiasm and most fruitful discussions, this work would not have been completed in time.

I am grateful to Dr R.K. Pandey for the useful discussions I had with him from time to time. The author is grateful to Professor Shashi K. Gulhati, the then Head of the Civil Engineering Department, I.I.T. Delhi for granting necessary funds for working at the India Meteorological Department Computer Centre, Lodhi Road, New Delhi. The author is thankful to Professor K.Seetharamulu, Professor T. Ramamurthy and Professor C.S.Surana for their constant affectionate encouragement during author's stay at this institute.

The author is thankful to the Department of Science and Technology, Government of Bihar for deputing him, for doing Ph.D. Financial assistance by I.I.T. Delhi is thankfully acknowledged.


(ii)

Thanks are also due to the staff of IMD Computer Centre, Lodhi Road, New Delhi. The author is greatly indebted to the kind cooperation of all the members of the I.I.T. Delhi Computer Centre.

I extend sincere thanks to my friends Mr M.Jothi, Mr F.Sinha, Mr A.N.Sinha and Mr D.S.Sidhu for helping me in preparation of the thesis.

Thanks are also due to Mr Harish Bhatia for neat typing of the thesis and Mr N.L.Arora for drawing the figures neatly.

Lastly I thank my children Anupam, Om, Archana and Alok and my wife Madhuri for their incessant patience during my preoccupation with this work.



Ram Narain Singh

ABSTRACT

This study concerns the elastic-plastic fracture analysis of thin sheets, of strain hardening materials following von-Mises yield criterion, using the finite element method. A versatile two dimensional computer program based on elastoviscoplastic approach has been developed for the purpose. The elastoplastic solution is obtained by continuing the time marching procedure in a series of pseudo time steps until the convergence is achieved. In each time step, incremental viscoplastic strains are computed. The finite element procedure uses quadratic isoparametric elements with 2x2 Gaussian integration.

Fracture parameters, for the first quadrant of a finite geometry centre-cracked specimen, with a sharp line crack, discretized into 74 elements and 259 nodes, have been determined, under uniaxial and biaxial loads, both in the linear elastic and elastic plastic fracture mechanics ranges. The effect of biaxiality on fracture parameters and plastic zone shape and size has been studied in detail.

The centre cracked specimen mentioned above is taken up for the study of fracture behaviour under creep. Path independent character of the Rice's C^* parameter is established. Time to crack initiation for various biaxiality ratios and under different applied loads is determined with critical Crack Tip Opening Displacement (CTOD) as criterion for crack initiation. Distribution of CTOD, equivalent stress and equivalent strain with time exposure is studied.

(v)

Stable Crack Growth (SCG) under plane stress condition has been simulated using an experimental uniaxial external load vs. crack length relation for a centre cracked finite geometry specimen, with first quadrant discretized into 98 quadratic isoparametric elements and 337 nodes. The size of the crack tip elements through which crack propagates is 1.33 percent of initial half crack length. Analysis is performed with crack advance under increasing external load and 'staircase' type of loading. Nodal release technique is used for crack propagation. Critical Crack Tip Opening Angle (CTOA), as criterion of crack initiation during SCG, is confirmed. The crack growth law proposed by Rice is confirmed and the values of the parameters for plane stress condition have been obtained.

Biaxial effects on SCG is studied with critical CTOA criterion for crack initiation and propagation. Some very interesting conclusions, in respect of J integral, plastic zone size, material constants, plastic energy and separation energy rates, are discussed.

CONTENTS

Chapter		Page
	CERTIFICATE	(i)
	ACKNOWLEDGEMENT	(ii)
	ABSTRACT	(iv)
	CONTENTS	(vi)
	LIST OF FIGURES	(xiv)
	LIST OF TABLES	(xix)
	NOTATIONS	(xx)
I	INTRODUCTION	1
II	REVIEW OF LITERATURE	10
	2.1 INTRODUCTION	10
	2.2 LINEAR ELASTIC FRACTURE MECHANICS	10
	2.2.1 Fracture Criteria	10
	2.2.2 Finite Element Method in LEFM	14
	2.3 ELASTIC-PLASTIC FRACTURE MECHANICS	15
	2.3.1 Fracture Criteria	16
	2.3.1.1 Finite element method in EPFM	22
	2.3.1.2 Biaxial load effects	24
	2.3.1.3 LSY effects	27

	2.3.2	Fracture Behaviour of Creeping Material	29
	2.3.3	Crack Growth Simulation	32
	2.3.3.1	Crack growth criteria	32
	2.3.3.2	Loading pattern and nodal release technique	36
	2.3.4	Stable Crack Growth Simulation under Biaxial Loading	38
	2.4	CLOSURE	39
III		MATHEMATICAL MODELING OF FRACTURE USING VISCOPLASTIC APPROACH	40
	3.1	INTRODUCTION	40
	3.2	MATHEMATICAL DEVELOPMENT	41
	3.2.1	Basic Concept and Rheological Models	41
	3.2.2	Mathematical Formulation	44
	3.2.2.1	Elastic strain determination	45
	3.2.2.2	Viscoplastic strain determination	46
	3.3	THE FINITE ELEMENT METHOD	52
	3.3.1	General Theory	52
	3.3.2	Choice of Elements and the Shape Functions	55
	3.3.3	Governing Equations	61
	3.3.4	Numerical Algorithm	63
	3.3.5	Stability Criteria	64
	3.3.6	Convergence Criteria	66

3.4	ANALYSIS OF FRACTURE PROBLEMS	68
3.4.1	J-integral	68
3.4.2	Plastic Energy	70
3.5	ANALYSIS OF CREEP PROBLEMS	72
3.6	CLOSURE	74
IV	DEVELOPMENT OF THE TWO DIMENSIONAL GENERAL PURPOSE FINITE ELEMENT PROGRAM	75
4.1	INTRODUCTION	75
4.2	ORGANISATION OF THE PROGRAM	76
4.3	DESCRIPTION OF THE PROGRAM	76
4.3.1	Data Processing	76
4.3.2	Elastic Plastic Analysis	81
4.3.2.1	Computation of stiffness matrix	81
4.3.2.2	Assembly of the load vector	82
4.3.2.3	Solution of equations	83
4.3.3	Computation of Fracture Parameters	84
4.3.4	Fracture Behaviour of Creeping Materials	85
4.3.5	Stable Crack Growth Simulation	85
4.4	SALIENT FEATURES OF THE PROGRAM	86
4.5	DIFFICULTIES IN HANDLING THE PROGRAM	88
4.6	DEFICIENCIES AND SUGGESTIONS FOR IMPROVEMENT	88
4.7	TEST EXAMPLE	89
4.8	CLOSURE	94

V	DETERMINATION OF FRACTURE PARAMETERS	95
5.1	INTRODUCTION	95
5.2	MATERIAL PROPERTIES, SPECIMEN GEOMETRY AND LOADING CONDITIONS	96
5.2.1	Material Properties	96
5.2.2	Specimen Geometry and Boundary Conditions	97
5.2.3	Loading Conditions	101
5.3	FINITE ELEMENT ANALYSIS	101
5.3.1	Discretization	101
5.3.2	Stability and Convergence Criteria	102
5.3.3	Fracture Parameters	102
	5.3.3.1 K, G, and J in LEFM	106
	5.3.3.2 J, COD and plastic energy in EPFM	107
5.4	RESULTS AND DISCUSSION	108
5.4.1	LEFM Range	108
5.4.2	EPFM Range	110
	5.4.2.1 One step and incremental step loading	110
	5.4.2.2 Path independence of J integral and biaxial load effects	110
	5.4.2.3 Plastic energy and biaxial load effects	116
5.5	STUDY OF PLASTIC ZONE SHAPE AND SIZE AND BIAXIAL LOAD EFFECTS	116
5.6	BIAXIAL LOAD EFFECTS ON COD	121
5.7	COMPUTING TIME	125
5.8	CONCLUSIONS	126

VI	FRACTURE BEHAVIOUR OF CREEPING MATERIALS	128
	6.1 INTRODUCTION	128
	6.2 MATERIAL PROPERTIES	129
	6.3 THEORY AND SOLUTION PROCEDURE	129
	6.4 CRITERION FOR CRACK INITIATION	130
	6.5 SPECIMEN GEOMETRY, THE LOADING AND THE BOUNDARY CONDITIONS	132
	6.6 FRACTURE BEHAVIOUR	132
	6.6.1 C* Parameter	132
	6.6.2 Time to Crack Initiation	133
	6.6.3 Equivalent Stress and Equivalent Strain Distribution	134
	6.7 RESULTS AND DISCUSSION	135
	6.7.1 Variation of CTOD with Time	135
	6.7.2 Study of the C* Parameter	142
	6.7.3 Equivalent Stress and Equivalent Stress Contour	143
	6.7.4 The Creep Curve and the Equivalent Strain Contour	148
	6.7.5 Effect of Creep Exponent	151
	6.8 CLOSURE	156
VII	STABLE CRACK GROWTH SIMULATION	157
	7.1 INTRODUCTION	157
	7.2 MATERIAL PROPERTIES AND SPECIMEN GEOMETRY	158
	7.3 MODEL FOR STABLE CRACK GROWTH	158

7.3.1	Discretization	160
7.3.2	Nodal Release Technique and Boundary Conditions	160
7.3.3	Load Stepping Technique and Computer Program	164
7.4	NON LINEAR STRAIN ENERGY RELEASE RATE AND SEPARATION ENERGY RATE	168
7.5	RESULTS AND DISCUSSION	170
7.5.1	J-Integral and its Relation with Crack Length	170
7.5.2	Crack Profile, the COD and the CTOD	172
7.5.3	Plastic Zone	178
7.5.4	Study of Relations Between COD, J and R	182
	7.5.4.1 CTOD vs. J Relation	182
	7.5.4.2 R vs. J relation	185
	7.5.4.3 Relation between incremental COD, incremental J and R	186
7.5.5	Stress Distribution	193
7.5.6	Strain Distribution	197
7.5.7	Plastic Energy	200
7.5.8	Study of Nonlinear Energy Release rate, separation energy rate and surface energy rate	200
7.6	COMPUTING TIME	203
7.7	CLOSURE	203

VIII	BIAXIAL LOAD EFFECTS ON STABLE CRACK GROWTH	204
8.1	INTRODUCTION	204
8.2	CRITERION FOR CRACK INITIATION AND STABLE CRACK GROWTH	205
8.3	MATERIAL PROPERTIES, SPECIMEN GEOMETRY, DISCRETIZATION AND NODAL RELEASE TECHNIQUE	206
8.4	LOADING PROCEDURE	206
	8.4.1 Incremental Steps for Uniaxial and Biaxial External Loading	207
	8.4.2 Boundary Conditions	207
8.5	FINITE ELEMENT PROGRAM	208
8.6	RESULTS AND DISCUSSION	209
	8.6.1 Load-Crack Size Relation	209
	8.6.2 Effects of Biaxial Loading on CTOD and CTOA	211
	8.6.3 J-integral	212
	8.6.4 Plastic Zone Shape, Size and Orientation	214
	8.6.4.1 Tension-tension (k=1)	214
	8.6.4.2 Uniaxial tension (k=0)	219
	8.6.4.3 Tension-compre- ssion (k=-1)	220
	8.6.4.4 Tension compre- ssion(k=-2)	220
	8.6.5 Study of Incremental CTOD($d\delta$) Incremental J(dJ) and Maximum Radius of Plastic Zone(R) Relation	221

	8.6.6	Stress Distribution	225
	8.6.7	Plastic Energy	232
	8.6.8	Separation Energy Rate	235
	8.6.9	Comparison of Parameters	239
	8.7	COMPUTING TIME	240
	8.8	CLOSURE	241
IX		CONCLUSIONS AND RECOMMENDATIONS	242
		REFERENCES	250
		APPENDIX I	A.1
		APPENDIX II	A.2