SPATIAL AND ANATOMICAL HARMONICS DOMAIN BASED BRAIN SOURCE LOCALIZATION

AMITA GIRI

DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MARCH, 2022

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by

AMITA GIRI

DEPARTMENT OF ELECTRICAL ENGINEERING

Submitted

in fulfillment of the requirements of the Degree of Doctor of Philosophy

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MARCH, 2022

Dedicated

To

My Beloved

Grandfather and Late Grandmother -

who always inspire me to dream big

Department of Electrical Engineering
Indian Institute of Technology Delhi
Hauz Khas, New Delhi, India-110016

Certificate

This is to certify that the thesis entitled **Spatial and Anatomical Harmonics Domain based Brain Source Localization**, submitted by **Amita Giri** to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** has been carried out under our supervision. The work contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree.

Prof. Lalan Kumar
Department of Electrical Engineering
and Bharti School of Telecommunication
Indian Institute of Technology Delhi
Hauz Khas, New Delhi, India - 110016.

Prof. Tapan K. Gandhi
Department of Electrical Engineering
Indian Institute of Technology Delhi
Hauz Khas, New Delhi, India - 110016.

IIT. Delhi
March 2022

Acknowledgment

First and foremost thanks to the almighty God for giving me the wisdom, determination and patience in exploring the wonderful world of Neuroscience.

I would like to express my deep and sincere gratitude to my advisor, Prof. Lalan Kumar for giving me the opportunity to work under his guidance. His vision, sincerity and motivation have been a source of inspiration to me. He always gave me the freedom to think and explore new research ideas. He has taught me the right way to present my research. His tireless hard work on me during my research enabled me to complete the research successfully. I am also deeply thankful to Prof. Tapan K. Gandhi, who co-supervised my Ph.D. research. He has always been approachable whenever I wanted to discuss my research ideas. I am truly grateful to his support for all these years.

I would like to thank the Ministry of Education, Government of India for providing financial support during the first year of my Ph.D. (2017-2018) and later upgrading it to the prestigious Prime Minister's Research Fellowship (PMRF) (2018-2022). I gratefully acknowledge the travel support from PMRF contingency fund, which has given me the wonderful opportunity to participate in national and international conferences, workshops, and present my work across the globe.

I would like to extend my heartfelt thanks to all my MSP labmates. In particular, I thank Deepika Kumari, Shiva Azimi, Anant Jain, Shivani Ranjan, Prerna Singh, and Ayush Tripathi for their incredible support at my workplace. I greatly appreciate my senior Soumya P. Dash, Jyoti Maheshwari, Rakhi Sharma and Soumyadip Banerjee, who have always given me best advice. Also, I would like to thank Divyanjali Sharma with whom I had some long thoughtful discussion about life and its purpose, especially during the evening walk. I feel extremely lucky and privileged to have so many wonderful friends.

Most importantly, I shall be forever grateful to my parents Mr. Gajendra Giri and Mrs. Pushpa Giri for supporting me with their heart and soul to pursue my academic career and have patiently nurtured me throughout my life. I would like to thank my brother Harsh Giri for his support and affection. Special thanks to Prof. Kumar Gaurav from NIT Uttarakhand and my uncle Dr. Ankik Giri, who inspired me to pursue Ph.D and turning direction of my life to research. Without their counseling, this thesis would not have been possible. I also wish to extend a special thanks to my friend Akshay Bhatt for his constant encouragement and unceasing loving support which made the completion of this piece of work possible.

At last, I would like to pay my sincere gratitude to one and all who have directly or indirectly helped me throughout this journey.

Amita Giri

Amita Giri

Abstract

Brain Source Localization (BSL) using Electroencephalogram (EEG) has been an active area of research because of its cost-effective and noninvasive nature. As the EEG data is spatially sampled over head, the subsequent localization performance is limited by the head-shape assumption for efficient data representation. In the literature, the human head is approximated by spherical shape. Hence, Spherical Harmonics (SH), the corresponding basis functions, have been the natural choice for EEG source reconstruction and localization. However, it requires more number of SH coefficients due to discontinuities at the boundary of the head. This thesis addresses the brain source localization problem in spatial and anatomical harmonics (spherical and head harmonics) domain. In particular, a novel set of basis functions called Head Harmonics (H^2) are developed to accurately represent the data sampled over head. The basis functions are formulated based on more realistic head dimension. The three spatio-temporal forward data models that include Infinite Homogeneous Isotropic Conductor (IHIC), Three Layer Concentric Spherical Head (TLCSH) and four shell, are presented with their respective gain matrix, orientation matrix and signal intensity matrix. A framework is presented to transform the spatio-temporal forward model to anatomical harmonics domain. The anatomical harmonics domain formulation leads to dimensionality reduction and increased contribution of source eigenvalues resulting in decreased computation and increased accuracy respectively. Subsequently, the inverse methods that include Minimum Variance Distortionless Response (MVDR), Multiple Signal Classification (MUSIC), Recursive MUSIC (R-MUSIC) and, Recursively Applied and Projected MUSIC (RAP-MUSIC) are formulated in H^2 domain for advanced BSL. The theory is additionally verified with real EEG data corresponding to epilepsy seizure, visual stimuli and mental arithmetic task.

For efficient BSL in the spatial domain, a novel Subspace Principal Vector Projection (SPVP) based approach is proposed that suppresses interference present in the activity state. SPVP method utilizes subspace correlation based mutual interference statistics and thus relaxes the strict spatial stationarity condition. In real time Brain Computer Interface (BCI) application, efficient interference suppression is advantageous to extract useful information from the raw EEG signals. The role of BSL in decoding the intended hand movement for

the potential BCI applications is additionally explored. As the EEG sensor records electrical activity from all cortical sources including region of no interest, that may be the reason for the poor performance in the sensor space. Classification of left and right hand movement from EEG signals in sensor and cortical source space (through BSL) is presented. It is to note that the conventional classification based BCI system controls external devices by providing discrete control signals to the actuator. A continuous kinematic reconstruction from EEG signal is better suited for practical BCI applications. Therefore, three novel source aware deep learning models i.e. Multi Layer Perceptron (MLP), Convolutional Neural Network - Long Short Term Memory (CNN-LSTM), and Wavelet Packet Decomposition (WPD) CNN-LSTM are proposed for Motion Trajectory Prediction (MTP). BSL is utilized for channel selection and accurate EEG trial selection in continuous trajectory prediction. Qualitative comparison of proposed model with state-of-the-art Multi-variable Linear Regression (MLR) model is presented.

The idea of utilizing anatomical basis for accurate EEG data representation is further extended for the representation of different anatomical shapes, including human scalp, skull, brain, tooth, face, lung, ventricle and brain cortical surfaces. Spherical harmonics are widely used for anatomical shape description. However, establishing a one-to-one correspondence between the object surface and the entire unit sphere may induce a large geometric distortion, in case the shape of the surface is too different from a perfect sphere. Thus, we proposed adaptive area-preserving parameterization methods for simply-connected open and closed surfaces. The key idea is to compute an adaptive area-preserving parameterization of any input surface onto the optimal spherical cap region of the unit sphere, which allows us to utilize the adaptive basis functions for the shape description and reconstruction.

सारांश

इलेक्ट्रोएन्सेफलोग्राम (ईईजी) का उपयोग कर ब्रेन सोर्स लोकलाइजेशन (बीएसएल) अपनी लागत प्रभावी और नोन-इनवेसिव प्रकृति के कारण अनुसंधान का एक सक्रिय क्षेत्र रहा है। चूंकि ईईजी डेटा सिर के ऊपर स्थानिक रूप से लिया जाता है, स्थानीयकरण प्रदर्शन कुशल डेटा प्रतिनिधित्व के लिए सिर के आकार की धारणा द्वारा सीमित होता है। साहित्य में, मानव सिर को गोलाकार आकृति द्वारा अनुमानित किया गया है। इसलिए, स्फेरिकल (गोलाकार) हार्मोनिक्स (एसएच), तदनुसार आधार फलन, ईईजी स्रोत पुनर्निर्माण और स्थानीयकरण के लिए स्वाभाविक पसंद रहे हैं। हालांकि, सिर की सीमा पर असंततता के कारण अधिक संख्या में एसएच गुणांक की आवश्यकता होती है। यह थीसिस स्थानिक और एनाटॉमिकल हार्मोनिक्स (स्फेरिकल और हेड (सिर) हार्मोनिक्स) डोमेन में मस्तिष्क स्रोत स्थानीयकरण समस्या को संबोधित करती है। विशेष रूप से, हेड हार्मोनिक्स (H^2) नामक आधार फलन का एक नया सेट विकसित किया जाता है जो सिर के ऊपर रिकॉर्डेड डेटा का सटीक प्रतिनिधित्व कर सके। आधार फलन को अधिक यथार्थवादी सिर के आयाम के आधार पर तैयार किया गया है। तीन स्थानिक-अस्थायी फॉरवर्ड डेटा मॉडल जिसमें इनफिनिट होमोजनोस आइसोट्रोपिक कंडक्टर (IHIC), थ्री लेयर कंसेंट्रिक स्फेरिकल हेड (TLCSH) और फॉर शेल शामिल हैं, उनके संबंधित गैन मैट्रिक्स, ऑरीएन्टैशन मैट्रिक्स और सिग्नल इन्टेन्सटी मैट्रिक्स के साथ प्रस्तुत किया गया है। स्थानिक-अस्थायी फॉरवर्ड मॉडल को एनाटॉमिकल हार्मोनिक्स डोमेन में बदलने के लिए रूपरेखा प्रस्तुत किया गया है। एनाटॉमिकल हार्मोनिक्स डोमेन फॉर्मूलेशन से आयामीता में कमी आती है और स्रोत अभिलाक्षणिक मान के योगदान में वृद्धि होती है जिसके परिणामस्वरूप क्रमशः कम गणना और सटीकता में वृद्धि होती है। तत्पश्चात्, इन्वर्स विधियाँ जिनमें मिनमम वेरीअन्स डिस्टॉर्शन रीस्पान्स (MVDR), मल्टीपल सिग्नल क्लासिफिकेशन (MUSIC), रिकर्सिव MUSIC (R-MUSIC) और, रिकर्सिवली अप्लाइड ऐन्ड प्रोजेक्टेड MUSIC (RAP-MUSIC) शामिल हैं, उन्नत बीएसएल के लिए H^2 डोमेन में सूत्रित किए जाते हैं। सिद्धांत को मिर्गी जब्ती, दृश्य उत्तेजनाओं और मानसिक अंकगणितीय कार्य के अनुरूप वास्तविक ईईजी डेटा के साथ भी सत्यापित किया गया है।

स्थानिक डोमेन में कुशल बीएसएल के लिए, एक उपन्यास सबस्पेस प्रिंसिपल वेक्टर प्रोजेक्शन (एसपीवीपी) आधारित दृष्टिकोण प्रस्तावित किया गया है जो गतिविधि क्षेत्र में मौजूद हस्तक्षेप को दबाता है। एसपीवीपी विधि सबस्पेस सहसंबंध आधारित पारस्परिक हस्तक्षेप आंकड़ों का उपयोग करती है और इस प्रकार सख्त स्थानिक स्थिरता की स्थिति को शिथिल करती है। वास्तविक समय में ब्रेन कंप्यूटर इंटरफेस (बीसीआई) अनुप्रयोग, कुशल हस्तक्षेप दमन ईईजी संकेतों से उपयोगी जानकारी निकालने के लिए फायदेमंद है। संभावित बीसीआई अनुप्रयोगों के लिए इच्छित हाथ आंदोलन को डीकोड करने में बीएसएल की भूमिका का अतिरिक्त रूप से पता लगाया गया है। चूंकि ईईजी सेंसर सभी कॉर्टिकल स्रोतों से विद्युत गतिविधि को रिकॉर्ड करता है, जिसमें बिना रुचि क्षेत्र भी है, यह सेंसर स्पेस में खराब प्रदर्शन

का कारण हो सकता है। सेंसर और कॉर्टिकल सोर्स स्पेस (बीएसएल के माध्यम से) में ईईजी संकेतों से बाएं और दाएं हाथ के आंदोलन का वर्गीकरण प्रस्तुत किया गया है। यह ध्यान देने योग्य है कि पारंपरिक वर्गीकरण आधारित बीसीआई प्रणाली एक्चुएटर को असतत नियंत्रण संकेत प्रदान करके बाहरी उपकरणों को नियंत्रित करती है। ईईजी सिग्नल से निरंतर कीनेमेटिक पुनर्निर्माण व्यावहारिक बीसीआई अनुप्रयोगों के लिए अनुकूल है। इसलिए, मोशन ट्रैजेक्टरी प्रेडिक्शन (एमटीपी) के लिए तीन उपन्यास स्रोत जागरूक डीप लर्निंग मॉडल, अर्थात् मल्टी लेयर परसेप्ट्रॉन (एमएलपी), कान्वलूशनल न्यूरल नेटवर्क - लॉन्ग शॉर्ट टर्म मेमोरी (सीएनएन-एलएसटीएम), और वेवलेट पैकेट डीकंपोजिशन (डब्ल्यूपीडी) सीएनएन-एलएसटीएम प्रस्तावित किये गए है। बीएसएल का उपयोग निरंतर प्रक्षेपवक्र आगम कथन में चैनल चयन और सटीक ईईजी परीक्षण चयन के लिए किया जाता है। अत्याधुनिक मल्टी-वेरीअबल लिनीअर रिग्रेशन (एमएलआर) मॉडल के साथ प्रस्तावित मॉडल की गुणात्मक तुलना प्रस्तुत की गई है।

सटीक ईईजी डेटा प्रतिनिधित्व के लिए एनाटॉमिकल आधार का उपयोग करने का विचार विभिन्न शारीरिक आकृतियों जैसे मानव स्कैल्प, खोपड़ी, मस्तिष्क, दांत, चेहरा, फेफड़े, वेंट्रिकल और मस्तिष्क कॉर्टिकल सतहों के प्रतिनिधित्व के लिए विस्तारित किया गया है। स्फेरिकल हार्मोनिक्स व्यापक रूप से शारीरिक आकार विवरण के लिए उपयोग किया जाता है। हालांकि, यदि सतह का आकार एक पूर्ण गोला से बहुत अलग होता है, तो वस्तु की सतह और पूरे इकाई क्षेत्र के बीच एक-से-एक पत्राचार स्थापित करने से एक बड़ी ज्यामितीय विकृति उत्पन्न हो सकती है। इसलिए, हमने सिम्पली-कनेक्टेड खुली और बंद सतहों के लिए अनुकूली क्षेत्र-संरक्षण पैरामीटराइजेशन विधियों का प्रस्ताव दिया है। हमारा मुख्य विचार इकाई क्षेत्र के इष्टतम गोलाकार कैप क्षेत्र पर किसी भी इनपुट सतह के अनुकूली क्षेत्र-संरक्षण पैरामीटराइजेशन की गणना करना है, जो हमें आकार विवरण और पुनर्निर्माण के लिए अनुकूली आधार फलन का उपयोग करने की अनुमति देता है।

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List of Abbreviations

AH	Adaptive Harmonics
ALP	Associated Legendre Polynomials
BCI	Brain Computer Interface
BEM	Boundary Element Method
BSL	Brain Source Localization
CAR	Common Average Referencing
CNN	Convolutional Neural Network
CNS	Central Nervous System
CSP	Common Spatial Pattern
DICS	Dynamic Imaging of Coherent Sources
ECoG	Electrocorticography
ERD	Event Related Desynchronisation
EEG	Electroencephalogram
ERS	Event Related Synchronisation
FDM	Finite Difference Method
FEM	Finite Element Method
fMRI	functional Magnetic Resonance Imaging
fNIRS	functional Near-InfraRed Spectroscopy
H^2	Head Harmonics
HSH	Hemispherical Harmonics
IES-MUSIC	Improved Sequential Multiple Signal Classification
IHIC	Infinite Homogeneous Isotropic Conductor
LAURA	Local Auto Regressive Average
LDA	Linear Discriminant Analysis
LORETA	Low Resolution Electrical Tomography
LSTM	Long Short-Term Memory
MEG	Magnetoencephalogram

ME	Motor Execution
MI	Motor Imagery
MLP	Multi Layer Perceptron
MLR	Multi-variate Linear Regression
MNE	Minimum Norm Estimates
MRCP	Movement Related Cortical Potential
MRI	Magnetic Resonance Imaging
MTP	Motion trajectory prediction
MVDR	Minimum Variance Distortionless Response
MUSIC	Multiple Signal Classification
R-MUSIC	Recursive Multiple Signal Classification
RMSE	Root Mean Square Error
RAP-MUSIC	Recursively Applied and Projected MUSIC
SFT	Spherical Fourier Transform
SH	Spherical Harmonics
SIR	Signal to Interference Ratio
sLORETA	standardized Low Resolution Electrical Tomography
SMR	Sensory Motor Rhythms
S-MUSIC	Sequential Multiple Signal Classification
SNR	Signal to Noise Ratio
SPVP	Subspace Principal Vector Projection
SVD	Singular Value Decomposition
TLCSH	Three Layer Concentric Spherical Head
WMNE	Weighted Minimum Norm Estimates
WPD	Wavelet Packet Decomposition

List of Symbols

$(.)^T$	Transpose of matrix or vector (.)
$(.)^H$	Conjugate transpose of matrix or vector (.)
$\mathbf{a}$	Manifold vector in spatial domain
$\mathbf{a_{nm}}$	Manifold vector in anatomical harmonics domain
$\mathbf{A}$	Manifold matrix in spatial domain
$\mathbf{A_{nm}}$	Manifold matrix in anatomical harmonics domain
A_n^m	Adaptive harmonics of order n and degree m
$\mathbf{B}$	EEG mode strength
$\mathbf{C}$	Adaptive harmonics coefficient matrix
$\mathbf{d}$	dipole moment vector
d_{area}	Area distortion
d_{angle}	Angle distortion
$\mathbf{e}$	Unit dipole moment orientation vector
I	Number of Sensors
$\mathbf{I}$	Identity matrix
$\mathbf{g}$	Gain vector in spatial domain
H_n^m	Head harmonics of order n and degree m
$\mathbf{M}$	Dipole moment orientation matrix
m	Degree of anatomical harmonics
n	Order of anatomical harmonics
N_a	Sensor array order
N_s	Number of time snapshots
P	Number of sources
$\mathbf{V}$	EEG potential in spatial domain
$\mathbf{V_{nm}}$	EEG potential in anatomical harmonics domain
P_n^m	Associated Legendre functions
$\mathbf{R_V}$	Data Covariance matrix

$\mathbf{r}$	Position vector
$\mathbf{S}$	Dipole signal intensity matrix
Y_n^m	Spherical harmonics of order n and degree m
$\mathbf{\Psi}$	Angular location of all sources
$\mathbf{\Phi}$	Angular location of all sensors
θ	Elevation angle
ϕ	Azimuth angle
$\mathbf{Z}$	White Gaussian noise matrix in spatial domain
$\mathbf{Z_{nm}}$	White Gaussian noise matrix in anatomical harmonics domain