

**A Study of Some Cryptographic Protocols
Based on
Discrete Logarithm Problem and Pairing**

By

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Department of Mathematics

*Submitted
in fulfillment of the requirements
of the degree of Doctor of Philosophy*

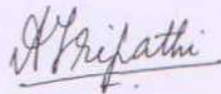
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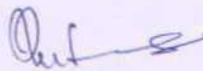
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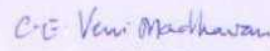
We are satisfied that the thesis entitled *A Study of Some Cryptographic Protocols Based on Discrete Logarithm Problem and Pairing* presented by Mr. INDIVAR GUPTA (2002RMA008) is worthy of consideration for the award of the degree of Doctor of Philosophy and is a record of the original bonafide research work carried out by him under our guidance and supervision and the results contained in it have not been submitted in part or full to any other University or Institute for award of any degree/diploma.



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Abstract

The notion of signcryption was introduced by Zheng [108] in 1997. The main objective was to achieve digital signature and encryption in a single logical step, instead of carrying out the two steps one after the other, as done traditionally, so that computation / communication cost is reduced significantly. This became more important when communication takes place in a distributed environment. Mu and Varadharajan [82] proposed a method for distributed signcryption and extended their idea to group signcryption. Kwak and Moon [64] proposed efficient distributed signcryption as group signcryption with sender identity confidentiality but the complexity of both the schemes depended on the number of users in the communicating group. The formal security proofs had also not been considered in any of these schemes.

Han and Yang [51] proposed a generic primitive called generalized signcryption. The main objective was to provide double functions when both confidentiality and authenticity are required simultaneously. It also aimed at providing single encryption or signature function when only confidentiality or authenticity is required (without any modification to the operation and additional computation). However, very little further work has been done in this area.

This thesis deals with cryptographic protocols based on discrete logarithm problem and pairing. Main protocols, which have been discussed in the thesis, include ‘distributed signcryption’ and its extension to ‘group signcryption’, ‘generalized signcryption’ and ‘generalized distributed signcryption’.

We introduce formal security notions of distributed signcryption and its extension to group signcryption for message confidentiality and unforgeability. As security proofs were not given by Mu and Varadharajan in their paper[82], we modify their scheme and give the proofs of security. We then propose two efficient distributed signcryption schemes and their extension to group signcryption on hyperelliptic curves in such a way that the complexity and communication

cost are independent of the number of users and weaknesses of existing schemes are overcome. Also a scheme for distributed signcryption from pairing has been designed satisfying the above properties. We address the issues of security of these schemes for message confidentiality and unforgeability.

We also address issues wherein designated groups in distributed signcryption or group signcryption are dynamically changing. Our main focus is on the issues of members leaving the group and new members joining the group. We have also addressed the other issues when either two groups merge or an existing group splits into two or more groups. Security issues have also been discussed.

In this thesis, we also study existing methods for generalized signcryption, ID based generalized signcryption and propose a new method for generalized signcryption. We extend the concept of generalized signcryption to introduce ‘generalized distributed signcryption’ and its further extension to ‘generalized group signcryption’. We also give methods for ‘generalized distributed signcryption’ and discuss their further extension.

In all the cases, where ever applicable, we give the security proofs in the random oracle model.

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