

**A THESIS ON
ON PROPAGATION OF DISCONTINUITIES IN
PHYSICAL GASDYNAMICS**

**By
MOHAN PRASAD
Department of Mathematics
Indian Institute of Technology
New Delhi**

**Submitted to the Indian Institute of
Technology, New Delhi for the award
of the Degree of Doctor of Philosophy
in Mathematics**

JULY, 1980

IN LOVING MEMORY OF MY DEPARTED MOTHER

C E R T I F I C A T E

This is to certify that the thesis entitled 'On Propagation of Discontinuities in Physical Gasdynamics' which is being submitted by Mr. Mohan Prasad for the award of Degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three and half years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

Rama Shankar

**(Rama Shankar)
Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-110016.**

ACKNOWLEDGEMENTS

I wish to express my deep sense of gratitude to Dr. Rama Shankar, Assistant Professor, Department of Mathematics, Indian Institute of Technology, Delhi for suggesting me the subject of this thesis and for his constructive guidance throughout the preparation of this thesis. I feel privileged to express my profound sense of gratitude to hon'ble Prof. U.N. Singh, Vice Chancellor, University of Allahabad for his generous help and keen interest in my work. It is really a matter of pleasure to acknowledge the suggestions given by Prof. C.P. Bhutani in this work and so I am highly grateful to him. I am also thankful to Prof. M.K. Jain and Prof. M.P. Singh for their constant encouragement throughout this work.

My thanks are due to Prof. H.L. Manocha, Head of the Department of Mathematics, Indian Institute of Technology, Delhi; to the authorities of Indian Institute of Technology, Delhi and the Council of Scientific and Industrial Research, India for providing me necessary research facilities and financial help to complete this work.

I will be failing in my duty if I don't gratefully acknowledge a word of inspiration from my wife for her lovely and pleasant behaviour inspite of her sacrifice and sufferings during the period of this work.

Finally I thank Mr. D.R. Joshi for his neat and clean typing.

Mohan Prasad
(Mohan Prasad)

S Y N O P S I S

This thesis entitled 'On Propagation of Discontinuities in Physical Gasdynamics' is divided into five chapters. Chapter I presents an Introduction to the thesis. Chapter II deals with the propagation of weak discontinuities in Physical Gasdynamics. Chapter III discusses some aspects of acceleration waves in dissociating gases. Chapter IV is devoted to study the geometry of streamlines in radiative and vibrational non-equilibrium gas flows and Chapter V is concerned with the study of the downstream effects of shock-waves in Non-equilibrium Magnetogasdynamics. The contents of the various chapters are outlined below:

CHAPTER I: INTRODUCTION

In this chapter, a brief historical review of the previous studies related to the present work and the set of Non-linear differential equations governing the flows are given.

CHAPTER II: PROPAGATION OF WEAK DISCONTINUITIES IN PHYSICAL GASDYNAMICS

In this chapter we have studied the propagation of weak discontinuities in various gaseous media. More specifically, we have discussed the formation of shock waves, that is, the termination of weak discontinuities into a shock in these

media. This chapter is divided into five sections. In section I we review the literature on this subject. The general theory for uniform as well as non-uniform propagation of weak discontinuities is presented in section II. The following sections present the application of the theory presented in section II to Magnetogasdynamics with dissipative mechanism^{*}, Non-equilibrium gasdynamics^{**} and radiation Magnetogasdynamics^{***}. It turns out that the role of electrical and thermal conductivities in the case of Magnetogasdynamic flow and dissociation (relaxation) in the case of non-equilibrium gas flow is to cause damping in the formation of the shock whereas radiation contributes to fast development of shock in radiating gases.

CHAPTER III: ON SOME ASPECTS OF ACCELERATION WAVES IN NON-EQUILIBRIUM DISSOCIATING GASES

This chapter deals with the propagation of acceleration waves in dissociating gases. A differential equation governing the local and global behaviour of the amplitude of the wave is formulated and solved. The dependence of wave amplitude on time is studied and its local and global behaviour with respect to growth and decay of the wave is discussed. The problem of breakdown of wave is also studied.

^{*}Int. J. Engng. Sci. Vol. 17 (1979), pp. 17-21

^{**}Appl. Sci. Res., 35 (1979) 1-10.

^{***}ZAMP, 30 (1979), 937-942

CHAPTER IV: ON THE GEOMETRY OF STREAM LINES IN RADIATIVE AND VIBRATIONAL NON-EQUILIBRIUM GAS FLOWS

This chapter discusses the geometry of stream lines in three dimensional steady flow of a radiative and vibrational fluids. The main purpose of this chapter is to investigate various dynamical and kinematical relations connecting the flow quantities with the geometrical parameters of the streamline trajectories. The expression for tangent, principal normal and binormal vectors and curvature and torsion of the streamlines are given in terms of velocity components, pressure, density, radiation and relaxation variables. It turns out that the radiating and relaxing character of the gas decreases the pressure gradient along the streamlines but the trajectories of the binormals lie on the surface of the constant pressure. Finally, the vorticity components are found in terms of the curvature of the streamlines and the velocity gradient along the streamlines, their principal normals and binormals.

CHAPTER V: SHOCK WAVES IN STEADY NON-EQUILIBRIUM MAGNETO- GASDYNAMICS

This chapter is devoted to study the downstream effects of the propagation of shock waves attached to the leading edge of an obstacle in Non-equilibrium Magnetogasdynamic-flow. More specifically, we have obtained the expression for curvature of the attached shock wave in such flows. It is shown that the attached shock wave in two-dimensional flow past a wedge becomes curved on account of non-equilibrium dissociation.

CONTENTS

CHAPTER

Page

SYNOPSIS

I

INTRODUCTION AND GENERAL SURVEY OF LITERATURE

1.	Introduction	1
(I)	Magnetogasdynamics with dissipative mechanism	1
(II)	Non-equilibrium gasdynamics	4
(III)	Non-equilibrium Magnetogas- dynamics and	14
(IV)	Radiation Magnetogasdynamics	15
2.	General Survey of Literature	17

II

PROPAGATION OF WEAK DISCONTINUITIES IN PHYSICAL GASDYNAMICS

1.	Introduction	31
	SECTION I	
2.	Survey of Literature	33
	SECTION II	
3.	General Theory for Uniform and Non-uniform Propagation of Weak Discontinuities	35
	SECTION III	
4.	Propagation of weak Disconti- nuities in Magnetogasdynamics in the Presence of Dissipative Mechanism	45
	SECTION IV	
5.	Propagation of Weak Disconti- nuities in Non-equilibrium Gasdynamics	53

SECTION V

6.	Formation of Shock Waves in Radiation Magnetogasdynamics	
(6.1)	Introduction	64
(6.2)	Equations of Motion	65
(6.3)	Velocity of Propagation	66
(6.4)	Growth and Decay of Weak Discontinuities Associated with the Wave Surface $Z(t)$ in a Uniform Medium	68
(6.5)	Non-uniform Propagation of Weak Discontinuities	72
	Appendix	78
III	ON SOME ASPECTS OF ACCELERATION WAVES IN NON-EQUILIBRIUM DISSOCIATING GASES	
1.	Introduction	80
2.	Coordinate System	83
3.	Growth Equations	89
	Summary	93
IV	ON THE GEOMETRY OF STREAM LINES IN RADIATIVE AND VIBRATIONAL NON-EQUILIBRIUM GAS FLOWS	
1.	Introduction	95
2.	Equations of Motion	96
3.	The basic Decomposition, Curvature and Torsion	99
4.	Gradients	101
5.	Vorticity Components	104
V	ON SHOCK WAVES IN STEADY NON-EQUILIBRIUM MAGNETOGASDYNAMICS	
1.	Introduction	107
2.	The Fundamental Equations	110
3.	Curvature of an Attached Shock Wave	111
	BIBLIOGRAPHY	122