ELECTROMAGNETIC WAVE PROPAGATION IN PLASMAS

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PREFACE

The propagation of electromagnetic waves in plasmas is an active field of research because of its many applications in science and technology. The results of investigations in this field are directly applicable to the communications problems associated with the propagation of radio waves in the ionosphere, rocket exhausts, interstellar matter and plasma sheaths around hypersonic vehicles in the atmosphere. These studies should also aid in the interpretation of the data obtained in connection with radio-probing of the ionosphere of the earth and other planets. Study of electromagnetic wave propagation is one of the most powerful tools in plasma diagnostics (the determination of the basic properties of plasmas); this phenomenon also provides an effective means of the study of band structure and collision processes in semiconductors. In the present thesis the author has investigated some interesting problems associated with the propagation of radio waves in plasmas and simple model semiconductors.

Recently it has been suggested that periodic variation of the propagation parameters in a plasma leads to amplitude modulation of an unmodulated electromagnetic wave interacting with it. The periodic variation in the propagation parameters can be caused by time varying changes in electron density and

temperature which may be caused by the presence of an acoustic wave (Sodha and Palumbo 1964) or the modulation of R.F. power in a discharge (John 1968). In the present work the author has proposed a new method for periodic variation of propagation parameters viz. the use of modulated magnetic fields. When an electromagnetic wave (of frequency ω) propagating along the direction of a modulated magnetic field ($B_0 + B_1 \exp(i\Omega t)$) interacts with a plasma, the side-bands of frequency $\omega \pm \Omega$ are generated; hence the transmitted and reflected waves are amplitude modulated. A theoretical analysis of this phenomenon has been presented which is applicable to plasmas as well as simple model semiconductors. This phenomenon has been suggested as a possible method of communicating with rockets and re-entry vehicles in the blackout region. It is seen that this mechanism is quite effective in communication with vehicles, provided that the signal, to be communicated from the vehicle, can be applied to the plasma sheath (around the vehicle in the re-entry phase) or the rocket exhaust (in the launch phase) in the form of a modulated magnetic field.

It is well known that when an amplitude modulated wave traverses an absorbing region in the ionosphere, its modulation depth may decrease. This phenomenon is known as demodulation and was observed experimentally amongst others by Aitchinson and Goodwin (1955), King (1959),

Cutolo et al. (1956) and Cutolo (1964). Some attempts have been made to explain the demodulation theoretically on the basis of self-interaction by Vilenskii (1953), Carlevaro (1956), Hibberd (1955), Ginzburg and Gurevich (1960), Sodha and Palumbo (1962) and Sodha and Kaw (1965). In the present work the author has presented a unified analysis of the demodulation of amplitude modulated waves in plasmas which incorporates the linear as well as nonlinear features of the earlier treatments.

The observations regarding the power dependence (summarized by Cutolo 1964) of the demodulation are unfortunately not in agreement and it seems that both power-dependent and power-independent types of modulation occur. The power-dependent modulation can be explained by considering the nonlinear nature of the plasma, while the explanation for power-independent demodulation should be sought in the dispersive nature of the medium or the selective fading (i.e. interference of two or many waves reflected from different points in the ionosphere). The following is a brief discussion of these mechanisms.

(i) Nonlinearity

When a moderately strong electromagnetic wave propagates in a plasma the conductivity becomes a function of the electric vector and hence the

complex amplitude of the electric vector at any point has a nonlinear dependence on the amplitude at some other point. Thus, if the initial wave is amplitude modulated, the coefficient of modulation will change as it interacts with the plasma. Vilenskii (1953), Hibberd (1955) etc. have invoked this mechanism to explain the phenomenon of demodulation.

(ii) Dispersion

The propagation parameters of the carrier wave and the two side bands are different due to the dispersive nature of the medium. This may also lead to the demodulation of the wave (Bailey 1956); the demodulation in this case is independent of the power of the wave. Sodha and Kaw (1965) have given an analytical theory based on this mechanism.

(iii) Selective fading

It was suggested by Boella (1954, 1955), Goodwin (1959) and Gurevich (1958) that the demodulation may be explained on the basis of selective fading, i.e. the interference phenomenon between two or many waves reflected from different points in the ionosphere. But the theoretical predictions based on this mechanism disagree with the experimental results (Cutolo 1964).

In the present work the author has theoretically investigated the phenomenon of demodulation of amplitude modulated waves in a plasma taking into account the effect of nonlinearity and dispersion simultaneously. The phenomenological approach as well as rigorous kinetic approach have been used to investigate this phenomenon. The treatment (phenomenological) is valid even for higher modulation frequencies ($\Omega >> \delta\nu$) because the form of the electric vector has been taken to be of the self-consistent; the earlier treatments of Vilenskii (1953), Hibberd (1955) and Sodha and Palumbo (1962) suffered due to this limitation. The numerical results obtained agree qualitatively with the experimental results of Cutolo (1964, 1968) and in the limiting cases ($\Omega << \delta\nu$) with the theoretical results of Sodha and Palumbo (1962). The self-interaction is also found to be responsible for the distortion of the modulation envelope, i.e. generation of second and third order side bands; these results also agree qualitatively with those cited by Ginzburg and Gurevich (1960). The change of modulation in the case of an amplitude modulated wave reflected back from the plasma free-space interface has also been studied. In the present day set up of high power transmitters the above analysis may have some interesting implications.

A plane polarized electromagnetic wave passing through an ionized medium, immersed in a steady magnetic

field, becomes elliptically polarized and its plane of polarization rotates; this phenomenon is commonly known as Faraday effect. In semiconductors, which are very much akin to plasmas as far as electronic transport phenomena are concerned (Shockley 1951), this effect is one of the easiest and most useful methods for studying the basic properties of the media (Moss 1962) viz. the band structure and the electron scattering mechanisms. The author has studied the effect of an external d.c. heating electric field on the Faraday effect in simple model one-band covalent and piezoelectric semiconductors taking the relevant scattering mechanisms into account. In the case of covalent semiconductors the results in the limiting cases of very high and very low heating fields reduce to the results of the two earlier workers - Subashiev (1965) and Gulyaev (1965) respectively. In another problem the author has studied the effect of the heating of electrons by the field of a moderately strong wave itself; the Faraday effect in such a case can be called as nonlinear Faraday effect.

The author has also investigated the phenomenon of linear and nonlinear wave propagation in a colloidal plasma, a plasma in which the electron density is controlled by fine dust of low work function material; competing processes of thermionic emission by and electron recombination with

suspended solid particles have been carefully considered. The expressions for d.c. conductivity and the complex conductivity corresponding to a wave-vector have been derived taking into account the collisions of electrons with neutral molecules as well as charged solid particles. Numerical calculations have been made for the attenuation and reflection coefficients and refractive index of such plasmas with different parameters. In the case of nonlinear wave propagation the effect of a moderately strong electromagnetic wave in changing the electron temperature and hence the field of the wave itself has been studied. It is found that in general the effect of nonlinearity is very small. These investigations may be of use for studying the electromagnetics of colloidal plasmas, as such plasmas are commonly encountered in rocket exhausts, hydrocarbon flames, interplanetary space and in closed cycle MHD power generators.

The thesis has been divided in four chapters whose titles and summaries are given below.

Chapter I: Amplitude Modulation of Electromagnetic Waves by Modulated Magnetic Fields.

In this chapter the author (Sodha et al. 1968) has developed a theory of the phenomenon that an electromagnetic wave passing through a plasma immersed in modulated magnetic field gets modulated in amplitude at the frequency of the alternating part of the magnetic field. The Boltzmann transfer equation for electron velocity distribution function

has been solved in the presence of the wavefields and a modulated magnetic field acting along the direction of propagation of the wave; various components of the distribution function, thus obtained, have been used to obtain the expression for the current density. The expression for the current density has then been substituted in the wave equation and wave equations corresponding to different frequency components have been solved; the only assumption being used is that the magnitudes of the generated side-band components are much smaller than the fundamental one. This analysis does not impose any restriction on various parameters such as electron collision frequency, wave frequency and magnitudes of the magnetic field. Different cases of practical interest have been considered and it is found that the phenomenon is appreciable. This phenomenon has also been studied in the case of simple model nondegenerate semiconductors. An interesting feature of the phenomenon is that the amplitude modulation in a wave propagating inside the plasma is, to a first approximation, proportional to the distance travelled.

This phenomenon has been suggested as a possible mechanism (Sodha and Arora 1968) for communicating with rockets and space vehicles in the blackout region. It is suggested that the signal to be communicated be applied in the form of modulated magnetic field across the plasma sheath around the re-entry vehicle or the rocket exhaust

(in the launch phase), then a wave reflected from such a plasma will be modulated in amplitude and thus the signal will be communicated to the ground. Some numerical calculations have been made and it is found that this phenomenon may be useful.

Chapter II: Nonlinear Demodulation of Amplitude Modulated Electromagnetic Waves in Plasmas.

This chapter presents an analytical investigation (Sodha et al. 1968, Ram and Arora 1967) of the phenomenon of demodulation of an amplitude modulated electromagnetic wave in a nonlinear dispersive plasma. The theory has been developed on the basis of phenomenological as well as rigorous kinetic approaches. In the phenomenological approach the electron momentum transfer equation and the energy balance equation have been solved simultaneously to obtain an expression for the nonlinear current density; the time dependent nature of the electron temperature and hence the electron collision frequency has been taken into account. This approach does not impose any restriction on the modulation frequency, collision frequency and the wave frequency but it is approximate in nature. The kinetic theory is more rigorous but it imposes severe restrictions on the modulation frequency ($\Omega << \delta\nu$). In the kinetic approach the Boltzmann transfer equation has been solved for electron velocity distribution function in the presence of a self-consistent field of an amplitude modulated wave. Various

expressions for the components of the distribution function give out the expression for nonlinear current density components.

The expressions for the nonlinear current density components obtained by the two approaches outlined above have been substituted in the wave equation. The resulting second order nonlinear differential equations have been solved for various field amplitudes by the method of successive approximations, with appropriate boundary conditions. The expressions for the electric vector have been used to study the change in modulation index of the wave. Some numerical calculations have been made in the case of both the approaches and it is found that the results obtained from kinetic approach agree with those obtained from the phenomenological approach. A high frequency electromagnetic wave propagating inside a plasma is found to be demodulated whereas a low frequency wave is overmodulated (the index of modulation increases). The demodulation of a high frequency wave decreases with increasing values of modulation frequency, electron density, collision frequency and the initial index of modulation. In the case of waves reflected back from a plasma boundary, overmodulation is obtained for high frequency waves (specially for low electron density plasmas) and demodulation for low frequency waves. From moderately high electron density

plasmas even the high frequency reflected wave is demodulated and this demodulation increases with increasing electron density and decreasing initial index of modulation. The magnitudes of generated side band components, which are responsible for the distortion of the modulation envelope, are significant for high initial indices of modulation and insignificant otherwise. Most of these results are in qualitative agreement with the earlier theoretical (Gurevich 1958, Ginzburg and Gurevich 1960) and experimental results (King 1959, 1957, Cutolo 1964, 1968).

Chapter III: Electromagnetic Wave Propagation in Semiconductors: Faraday Effect

In this chapter the author (Arora 1968, Arora and Gupta 1968) has studied the effect of the heating of electrons of the medium on the Faraday rotation and ellipticity; the heating of electrons may be because of external d.c. electric field or the field of the wave itself. The well known expressions for the Faraday rotation and ellipticity have been used which are written in terms of conductivity tensor derived on the basis of kinetic approach. In the case of covalent semiconductors at room temperatures deformation potential scattering has been considered and the distribution of electron velocity derived by Yamashita and Watanabe (1954) has been used while averaging the complex conductivity. It is found

that the Faraday rotation and ellipticity decrease with increasing value of the heating field. To study the effect of heating fields in piezoelectric semiconductors at low temperatures both the deformation potential scattering (DPS) and piezoelectric potential scattering (PPS) have been taken into account simultaneously (Onuki and Shiga 1966) and the distribution function derived from the Boltzmann transfer equation. It is seen that in this case also the Faraday rotation and ellipticity decrease with increasing value of the heating field; rotation and ellipticity are also found to decrease with increasing value of scattering parameter (DPS/PPS). A special feature of this variation is that when PPS dominates over DPS the rotation and ellipticity are not much affected by the heating field and this is because the distribution function becomes Maxwellian.

In another problem the author has studied the effect of electron heating by the field of the wave itself. Solutions of the nonlinear wave equations governing the wave propagation in semiconductors, similar to those derived by Sodha and Palumbo (1963) in the case of plasmas, have been used for studying the Faraday rotation and ellipticity. It is found that the contribution of the nonlinearity is significant in some cases when the linear rotation is small but in most of the cases it comes out to be insignificant.

Chapter IV: Wave Propagation in Colloidal Plasmas

The author (Sodha et al. 1968) has also studied

linear and nonlinear wave propagation in a colloidal plasma taking the collisions of electrons with neutral molecules and charged solid particles into account. In contrast to the phenomenological approach used by Sodha et al. (1963), the author in the present investigation has used the expressions for the complex conductivity, derived on the basis of kinetic theory, to calculate the values of refractive index, absorption and reflection coefficients. The conductivity is found to increase with increasing the temperature of the plasma, size and the concentration density of suspended solid particles.

In the study of nonlinear wave propagation, the phenomenological approach is used, i.e. the nonlinear current density has been obtained by solving the momentum transfer and energy balance equations simultaneously. The second order nonlinear differential equations have been solved by the method of successive approximations. Numerical calculations show that the contribution of the nonlinearity is very small in colloidal plasmas; the reason is that the transfer of energy acquired by the electrons from the field to solid particles is very effective and this does not let the electron temperature rise significantly above the gas temperature, thus the nonlinearity cannot be large.

The work in the present thesis has resulted in the following publications:

1. Amplitude modulation of electromagnetic waves by alternating magnetic fields, M.S. Sodha, A.K. Arora and P.K. Kaw, Proc. Phys. Soc. (GB) (J. Phys. A), Ser. 2, 1, 89 (1968).

2. Amplitude modulation of electromagnetic waves by alternating magnetic fields: Modified theory and application, M.S. Sodha and A.K. Arora (communicated) (1968).

3. Nonlinear demodulation of an amplitude modulated wave in a plasma, Sant Ram and A.K. Arora, Can. J. Phys., 45, 2543 (1967).

4. Nonlinear demodulation of an amplitude modulated wave in a plasma: Elementary approach, M.S. Sodha, A.K. Arora and A.K. Aggarwal, Radio Science (Dec. 1968).

5. Hot electron Faraday effect in semiconductors, A.K. Arora, Brit. J. Appl. Phys. (J. Phys. D), Ser. 2, 1, 521 (1968).

6. Microwave free carrier Faraday effect in piezoelectric semiconductors, A.K. Arora and B.M. Gupta, Brit. J. Appl. Phys. (J. Phys. D), Ser. 2, 1, Dec. 1968.

7. Electromagnetic wave propagation in colloidal plasmas, M.S. Sodha, S. Guha and A.K. Arora (communicated) (1968).

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TABLE OF CONTENTS