

# LAMINAR BOUNDARY LAYERS WITH HEAT AND MASS TRANSFER

by  
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**DOCTOR OF PHILOSOPHY**



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INDIA  
1989

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MY PARENTS ...

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Research Scholar, Mathematics Department to the Indian  
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under my guidance and supervision and has fulfilled  
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## ABSTRACT

The work presented in this thesis entitled "LAMINAR BOUNDARY LAYERS WITH HEAT AND MASS TRANSFER" is divided into six chapters. The development of the subject, the motivation of the thesis and the necessary mathematical tools are presented in Chapter I.

In Chapter II the laminar compressible forced convection boundary layers over flat plate and two-dimensional and axisymmetric bodies with heat transfer and vectored mass transfer are studied. For the flat plate case, the viscous dissipation effects are included. With the help of the similarity transformations, the governing system of PDEs are reduced to the system of ODEs which is then solved by employing the Keller-Box finite-difference method. From the computed solutions, the influence of vectored mass transfer on velocity, skin-friction, enthalpy and heat transfer are observed.

Chapter III concerns with the laminar forced convection nonsimilar boundary layers over slender elliptic cylinders of arbitrary shape with vectored mass transfer and viscous dissipation. The study is made subject to both isothermal wall and the constant

heat flux wall conditions. Using certain transformations the governing PDEs of the problem are reduced to a computationally convenient form of PDEs which are then solved by employing the Keller-Box finite-difference method. The substantial influence of the body shape and the vectored mass transfer on the point-of-separation, the skin-friction and the heat transfer are observed.

In Chapter IV, the effects of radiation interaction in free convection laminar boundary layer flows over isothermal vertical and horizontal surfaces are studied. The fluid is assumed to be gray, emitting and absorbing but non-scattering and the study is made in the optically thick radiation limit. With the help of new set of similarity transformations, the governing system of PDEs are first reduced to a system of ODEs, which is then solved by employing the quasilinearization technique. From the numerical results it is found that the effects of radiation are nonlinear and the increase in radiation interaction increases the shear stress and the heat transfer.

Chapter V deals with the laminar natural convection boundary layer of air over an isothermal vertical plate with the normal carbondioxide injection. The

boundary layer flow which is two-component in nature is studied after including thermal-diffusion and diffusion-thermo effects. Using the similarity transformations, the system of governing nonlinear coupled PDEs are reduced to the system of nonlinear coupled ODEs. This system of ODEs is then solved with the help of the quasilinearization technique. From the quasilinearized solutions it is found that the increase in the carbondioxide injection decreases the skin-friction and the heat transfer and the inclusion of thermal-diffusion and diffusion thermo effects enhances the heat and mass transfer. The single-component results obtained via the present investigation are also presented.

The laminar forced convection thermo-micropolar boundary layer flow over an isothermal or constant heat flux horizontal plate with vectored mass transfer is studied in Chapter VI. The microinertia of the fluid is assumed to be a variable. Also the viscous dissipation effects are included in the isothermal wall case. The reduced system of nonlinear ODEs are handled with the help of quasilinearization algorithm. The effects of the micropolar parameter and the vectored mass transfer on velocity, gyration, temperature, shear stress and heat transfer are examined.

*Summary and conclusions of the thesis and a discussion on numerical methods for separating boundary layers are provided respectively in Appendices I and II.*

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