REDUCED ORDER MODELLING AND FAST SIMULATION STRATEGIES FOR NONLINEAR DYNAMICAL SYSTEMS

by

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Certificate

This is to certify that the thesis entitled **"Reduced Order Modelling and Fast Simulation Strategies for Nonlinear Dynamical Systems"**, submitted by **Shahkar Ahmad Nahvi** to the Indian Institute of Technology Delhi, for the award of the degree of Doctor in Philosophy in Electrical Engineering, is a record of the original, bona fide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other university or institute for the award of any degree or diploma.

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Abstract

Dynamical systems are used to model diverse physical and artificial processes. Realistic and accurate description of many such systems tends to be of large dimensions, and their simulation is not possible without expenditure of considerable amounts of computational resources and time. Approximation is thus crucial for cost-effective simulation. Model order reduction (MOR) techniques help realize this objective in a computationally efficient way. They result in a dimensionally reduced system with input-output mapping similar to the large system being approximated.

MOR methods for large Linear time invariant systems are well researched and have been extensively covered in literature. Many such techniques have been extended to nonlinear systems, but research in nonlinear model reduction is in its infancy. Though strongly motivated and having a broad scope of applications, nonlinear model reduction also poses a different set of problems to be solved. This thesis represents an effort in developing better reduced order modelling techniques for nonlinear dynamical systems.

The focus in this thesis is on two popular techniques for nonlinear model reduction, Proper Orthogonal Decomposition (POD) and the Trajectory piece-wise linear (TPWL) approximation. It aims at improving the applicability and computational performance of these strategies and the approximation qualities and robustness of the reduced-order models obtained using them.

Although TPWL is a popular and widely applied tool for nonlinear model reduction, recent works have shown that TPWL models can under-perform for some applications. Questions have also been raised about the heuristic choices to be made during the course of the TPWL procedure. The endeavour in this work is to study the efficacy of the TPWL approximation, and this involves investigating the implicit assumptions, the dependence on heuristics and the applicability of

the TPWL method. Another point of focus is the method of basis extraction in POD, which constitutes a significant part of the total computational cost of the model reduction procedure.

In TPWL, in the offline stage, linearisations on the nonlinear system trajectory are done and stored. A superposition strategy interacts with this database to approximate the nonlinear system dynamics in the online phase. In this thesis, new methods are introduced to improve both these stages of the TPWL procedure. For linearisation point selection, new strategies that seek to address the important concerns regarding previously reported methods are proposed and validated. For the online phase, a new strategy is proposed for improving the superposition of linear systems in TPWL. A separate issue investigated in TPWL is its applicability to a wider class of nonlinear systems, the issues arising out of which are analysed, and in the process, a new method for nonlinear model reduction is introduced. Finally, a new alternative to extract basis functions for POD is illustrated and it is shown that the need for computationally heavy, a-priori simulations of the nonlinear system can be circumvented.

The organisation of this thesis is as follows, Chapter-1 consists of the introduction to the thesis, it explains the motivation and builds the background for the subsequent chapters. Chapter-2 contains a review of important works in the field of MOR, with emphasis on nonlinear dynamical systems. Chapter-3 deals with the issue of linearisation point selection in TPWL. Chapter-4 investigates the limitations of conventional superposition schemes in TPWL and in Chapter-5 the applicability of TPWL to more general nonlinear systems is analysed. Chapter-6 deals with POD and proposes the strategy to reduce the computational costs of the POD procedure. Finally, in the last chapter a summary of the thesis is presented and future directions of research are identified.

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