

**DEMODULATION AND CROSS MODULATION OF
ELECTROMAGNETIC WAVES IN
PLASMAS**

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PREFACE

The propagation of electromagnetic waves through an ionized medium has attracted a great deal of attention because of its manifold applications in different branches of science and technology. The characteristic feature exhibited by the propagation of a moderately strong wave in a plasma is the appearance of nonlinear effects which lead to harmonic generation, demodulation and cross modulation. These nonlinear phenomena are, in general, encountered in the propagation of radio waves in the ionosphere, communications through rocket exhausts and plasma sheaths around re-entry vehicles. Moreover, the study of the electromagnetic wave propagation is one of the most powerful tools for plasma diagnostics, i.e. the determination of the basic plasma parameters. The present thesis pertains to the study of some aspects of the phenomena of demodulation, gyrodemodulation and cross modulation; the prefix "gyro" means that the frequency of the carrier wave is equal to the gyrofrequency of the electrons.

When an amplitude-modulated wave traverses an absorbing region of the ionosphere, its index of modulation or the modulation depth decreases. This

phenomenon is known as demodulation and has been observed experimentally by Atchinson and Goodwin (1955), Cutolo et al (1956), King (1959) and Cutolo (1964). Some attempts have also been made to explain the phenomenon of demodulation theoretically on the basis of self-interaction by Vilenskii (1953), Carlevaro (1956), Hibberd (1955), Ginzburg and Gurevich (1960), Sodha and Palumbo (1962) and Sodha and Kaw (1965). In the present work, the author has presented a fairly rigorous analysis using the kinetic approach of the phenomenon of demodulation of an amplitude-modulated wave in a plasma, which incorporates the linear as well as nonlinear mechanisms simultaneously. As no conclusive experiments regarding the dependence or independence of the phenomenon of demodulation on the power of the incident wave have been reported upto date, two different mechanisms (linear and nonlinear) have been invoked to explain the phenomenon of demodulation. Ginzburg and Gurevich (1960) have suggested that both of these mechanisms may contribute significantly to the observed demodulation. The following is the brief discussion of linear and nonlinear mechanisms:

(a) Nonlinear Mechanism:

When a moderately strong electromagnetic

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wave propagates through a plasma (ionosphere), the conductivity becomes a function of the electric vector and hence the amplitude inside the ionized medium has a nonlinear dependence on the amplitude of the wave at the plasma boundary. Thus, if the initial wave is amplitude-modulated, then the coefficient of modulation will change as it reacts with the ionized medium. The phenomenon of demodulation has been explained on the above basis by Vilenskii (1953), Hibberd (1955) and King (1959).

(b) Linear Mechanism:

Linear demodulation is caused either by dispersion or by selective fading and is independent of the power of the wave.

(i) Dispersion:

The propagation parameters of the carrier wave and the two side bands are different due to the dispersive nature of the medium. This may lead to the phenomenon of demodulation as has been suggested by Bailey (1956) and investigated analytically by Sodha and Kaw (1965). The demodulation in this case

is independent of the power of the wave.

(ii) Selective Fading:

Boella (1954, 1955), Goodwin (1959) and Gurevich (1958) have suggested that the phenomenon of demodulation can also be explained on the basis of selective fading, i.e. the interference phenomena between two or many waves reflected from different points in the ionosphere. The theoretical predictions based on this mechanism are not in accordance with the experimental results of Cutolo (1964).

In Chapter I, the author has investigated analytically the phenomenon of demodulation of an amplitude-modulated wave in a plasma taking into account the effect of nonlinearity and dispersion simultaneously. Use of the kinetic approach (involving the techniques of Boltzmann's transfer equation) has been made for studying this phenomenon. The results obtained are in accordance with the theoretical results of Sodha and Palumbo (1962). The phenomenon of demodulation is also found to be responsible for the distortion of the modulation envelope, i.e. generation

of the second and the third order side bands; these results are also in agreement with those cited by Ginzburg and Gurevich (1960). The phenomenon of demodulation has also been studied for the case when the wave is reflected back into free space. Some numerical results have also been presented in the discussion for both the cases.

The phenomenon of demodulation is found to be very much pronounced when the frequency of the wave coincides with the gyrofrequency of the electrons. Cutolo et.al. (1956) concluded that the demodulation at gyroresonance may be quite significant even at low powers of the transmitting station (~ 3 KW). Thus in the medium wave transmission of radio waves through the ionosphere, the effect of earth's magnetic field plays a very important role. The phenomenon of gyro-demodulation is of great interest for radio wave scientists and a lot of experimental as well as theoretical work has been done on this aspect. Cutolo (1951, 1956, 1964), Mitra (1954) and Atchinson and Goodwin (1955) have made the experimental observations. Hibberd (1956) and King (1959) have tried to explain the phenomenon theoretically. These workers have analysed the phenomenon of gyro-demodulation by the phenomenological

approach and used the geometrical optics approximation. Sodha and Palumbo (1963) have given the kinetic treatment. All these treatments are limited to low modulation frequency only.

In Chapter II, the author has investigated analytically the phenomenon of gyro-demodulation by the kinetic approach when an amplitude-modulated wave propagates along the direction of the magnetic field, taking into account the effect of nonlinearity and dispersion of the medium simultaneously.

The phenomenon of demodulation at gyro-resonance of an amplitude-modulated wave reflected back into free-space from the plasma boundary has also been investigated.

Some numerical results have also been presented in the discussion, illustrating the variation of gyro-demodulation with relevant plasma parameters.

In 1937, Bailey and Martyn proposed a theory of the ionospheric cross modulation along the following lines. Radiation emitted by the disturbing transmitter is strongly absorbed in a region of the ionosphere where the wanted wave is refracted and attenuated. Absorption of the disturbing radiation raises the temperature of

the ionosphere. If the amplitude of the disturbing radiation varies with time, so will the resulting temperature and hence attenuation suffered by the wanted wave. A review of this work has been given by Bailey (1956). In the simplest experimental arrangement, an initially unmodulated wave of carrier frequency ω_1 (the wanted wave) traverses a region of the ionosphere in which a second wave at a different carrier frequency ω (the disturbing wave) is heavily absorbed. The disturbing wave is amplitude-modulated at a definite modulation frequency Ω . On reception, the frequency of the wanted wave is found to be modulated at the same modulation frequency Ω , also at twice this modulation frequency 2Ω .

Hibberd (1957), Ginzburg and Gurevich (1960) and Bailey (1965) have studied the phenomenon of cross modulation by the phenomenological approach. Caldirola and Barbieri (1965) and Layzer and Menzel (1965) have given the kinetic treatments. All these workers have made use of the geometrical optics approximation for obtaining the field amplitudes. Moreover, all these workers have taken the wanted wave to be weak, i.e. they have neglected the effect of the wanted wave on the collision frequency while the experiments have

also been conducted when the wanted wave is as strong as the disturbing wave (Huxley 1950, 1952).

In Chapter III, the phenomenon of the first and the second order cross modulations has been investigated analytically by the kinetic approach taking dispersion into account. The wanted wave has also been taken to be strong, thereby, the effect of the wanted wave on the collision frequency.

The phenomenon of cross modulation has also been studied when wave is reflected back into free space from the plasma-free space interface.

Some numerical results have also been presented in the discussion for both the cases.

The thesis has been divided into three chapters, the titles and summaries of which are given below:

Chapter I: Nonlinear Demodulation of an Amplitude-Modulated Electromagnetic Wave in a Plasma:

In this chapter, the author (Ram and Arora, 1967) has presented an analytical investigation of the phenomenon of demodulation when an amplitude-modulated electromagnetic wave propagates in a nonlinear dispersive ionized medium (plasma). The theory has been

developed on the basis of the kinetic approach. The theory presented is valid only for low modulation frequencies ($\Omega \ll \delta \omega$). In the kinetic approach, the Boltzmann transfer equation has been solved in order to obtain the electronic velocity distribution function in the presence of the electric vectors of the carrier and the two side bands. These expressions for the electronic velocity distribution function give the expressions for the nonlinear current density components.

The expressions for the nonlinear current density components obtained in a manner outlined above have been substituted in the wave equation. The resulting system of the second order nonlinear differential equations has been solved for the various field amplitudes by the method of successive approximations with appropriate boundary conditions. These expressions for the electric vectors have been used to study the phenomenon of demodulation. Some numerical results have been also presented in the tabular form and it is found that in the case of propagation inside the plasma, the demodulation decreases with the increasing electron density, initial index of modulation and electron collision frequency. In the case of waves

reflected from the plasma boundary, the demodulation is found to increase with the increase of the electron density and the decrease of the initial index of modulation. Moreover, the calculations have been also performed for the generated side band components but not presented because their magnitudes have been found to be insignificant except for high initial indices. These generated side band components are responsible for the distortion of the modulation envelope. These results agree with the results cited in the literature by Ginzburg and Gurevich (1960).

Chapter II: Nonlinear Demodulation of an Amplitude-Modulated Electromagnetic Wave Propagating in a Magnetoplasma:

This chapter (Ram and Kaw, 1967) presents the study of the phenomenon of demodulation in the presence of an external magnetic field when an amplitude-modulated electromagnetic wave propagates along the direction of the external static magnetic field into a nonlinear dispersive ionized medium. This phenomenon of demodulation has been studied by the kinetic approach at gyroresonance, i.e. when the frequency of the carrier wave is equal to the gyrofrequency of the electrons in the presence of an external magnetic field. The Boltzmann transfer equation for electrons is solved in the

presence of the electric vectors of the carrier and the two side bands and thus, enabling the expressions for the electronic distribution function to be evaluated and thereby, allowing the expressions for the nonlinear current density components to be written.

The expressions for the nonlinear current density components have been substituted in the wave equation and solved by the method outlined in Chapter I in order to obtain the various field amplitudes. These expressions of the amplitudes have been used to investigate the phenomenon of gyro-demodulation. Some numerical results have been also presented in the tabular form and it is concluded that the percentage gyro-demodulation in the case of the wave propagating inside a magnetoplasma decreases with the increase of the initial index of modulation, electron density and the decrement is very sharp when the strength of the electric field is reduced. In the case of the wave reflected from the magnetoplasma-free space interface, regarding the variation of the percentage gyro-demodulation, the same conclusions can also be drawn. The percentage gyro-demodulation in the case of the reflected wave is found to be slightly more than that of the wave propagating inside the magnetoactive medium. The

numerical results are in good agreement with the theoretical results of Ginzburg and Gurevich (1960) as well as with the experimental results of Cutolo (1964).

Chapter III. Cross Modulation in a Nonlinear Dispersive Plasma:

In this chapter, the author (Ram, 1969) has investigated analytically the phenomenon of the first and the second order cross modulations by the rigorous kinetic approach taking dispersion into account when both the wanted and disturbing waves are equally strong. The disturbing wave is modulated in amplitude at some definite modulation frequency and consists of the electric vectors of the carrier and the two side bands while the wanted wave is unmodulated. In the kinetic approach, the Boltzmann transfer equation for the electrons is solved in the presence of the above mentioned electric vectors and the electronic velocity distribution function is determined by taking into account the time dependences of the isotropic and anisotropic parts of the distribution function and thus, the expressions for the nonlinear current density components are evaluated.

The expressions for the nonlinear current density components have been substituted in the wave

equation and the solutions of various field amplitudes are obtained by the perturbation technique under appropriate boundary conditions. These various field amplitudes are used to study the phenomenon of the first and the second order cross modulations. Some numerical results have also been presented and it is seen that in the case of the wave propagating inside the plasma, the percentage cross modulations increase with the increase of the initial index of modulation and decrease with the increase of the electron density and collision frequency. In the case of the wave reflected from the plasma-free space interface, the percentage cross modulations increase with the initial index of modulation, electron density and collision frequency. The magnitude of the second order cross modulation is found to be small in comparison to the first order cross modulation in either cases. These results are in good agreement both qualitatively and quantitatively with the theoretical and experimental results of Ginzburg and Gurevich (1960) and Bailey (1965).

The work reported in the present thesis has resulted in the following publications:

- (i) Ram, S. and Arora, A.K., Nonlinear Demodulation of an Amplitude-Modulated Wave in a Plasma, Can. J. Phys., 45, 2543-2559, (1967).
- (ii) Ram, S. and Kaw, P.K., Nonlinear Demodulation of an Amplitude-Modulated Electromagnetic Wave Propagating in a Magnetoplasma, Can. J. Phys., 45, 3991-4010, (1967).
- (iii) Ram, S., Cross Modulation in a Nonlinear Dispersive Plasma, Radio Science (URSI, USA), No. 2, 4, (Feb., 1969).

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