Differential Flatness of Open and Closed-loop Kinematic Chains

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CERTIFICATE

This is to certify that the thesis titled **Differential Flatness of Open and Closed-loop Kinematic Chains** being submitted by **Sasanka Sekhar Sinha** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy** is a bona-fide record of the research work done by him under our supervision in conformity with the rules and regulations of the institute.

The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any Degree or Diploma.

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Sasanka Sekhar Sinha

ABSTRACT

Under-actuated mechanical systems come with the notable benefit of cost reduction compared to their fully actuated counterparts due to fewer actuators, and often result in energy efficient designs. The drawback of such systems is that planning and controlling trajectories requires advanced formulation. Flatness-based approach however facilitates implementation of trajectory planning and tracking. Planning is reduced to an algebraic enumeration and a feedback controller to track the flat output trajectories can be designed readily.

Having said all these, proving flatness and finding flat outputs for a system is not trivial. Till date, only necessary or sufficient conditions are available to test flatness. Flatness being intimately tied to the system structure requires one to exploit the physical arguments in the search of flat outputs. The fundamental question that we try to answer in this work is: what mechanical elements in a kinematic chain can be altered or added in which manner so that the resultant system is amenable to being under-actuated by being differentially flat. These modifications range from system mass-inertia parameters to added counter-balancing masses or adding passive mechanical elements like springs and dampers.

This work reports elementary systems consisting of single rigid bodies in 2-D space and go on to show that the flatness property of complex kinematic chains can often be inferred from recursive examination of flatness of its smaller sub-units. In this process, we have a new understanding of the flatness demands of mechanical systems which are modeled as connected rigid bodies. Most mechanical systems are shown to be either configuration flat or at the most depend on the first derivative of the variables (velocity). We have proposed new control affine systems wherein torques and forces enter the set of flat outputs.

Closed chain mechanisms are suitable candidates for high speed applications which require low inertia. In this thesis, we explore classes of binary closed-chain differentially flat systems and propose a systematic approach to flattening them. We demonstrate that by carefully choosing mass distribution and adding passive mechanical elements like springs, an under-actuated closed-loop system can be rendered differentially flat. Numerical simulation of trajectory planning for such systems demonstrates the effectivness of the strategy.

Most interestingly, differential flatness is capable of providing feasible trajectories in systems which are non flat, but allow a flat approximation. We introduce a weaker form of differential flatness for mechanical systems which has been termed as 'partial differential flatness' in some literature. Mostly, systems with cyclic or ignorable coordinates exhibit such a property. Under this category, differential flatness of floating chain and snake-like robots is also established.

सार

कम एक्चुएटर के कारण अपने पूरी तरह से सक्रिय समकक्षों की तुलना में अंडर–एक्ट्यूएटेड मैकेनिकल सिस्टम कम कीमत के उल्लेखनीय लाभ के साथ आते हैं, और अक्सर ऊर्जा कुशल डिजाइन में परिणाम होते हैं। ऐसी प्रणालियों का दोष यह है कि प्रक्षेपवक्रों की योजना बनाने और नियंत्रित करने के लिए उन्नत सूत्रीकरण की आवश्यकता होती है। फ्लैटनेस–आधारित दृष्टिकोण हालांकि प्रक्षेपवक्र योजना और ट्रैकिंग की सुविधा प्रदान करता है। योजना को सरल गणना तक सीमित कर दिया गया है और फ्लैट आउटपुट प्रक्षेप पथ को पालन करने के लिए एक फीडबैक कंट्रोलर को आसानी से डिजाइन किया जा सकता है।

फिर भी, किसी सिस्टम के लिए फ्लैटनेस साबित करना और फ्लैट आउटपुट खोजना आसान बात नहीं है। आज तक, फ्लैटनेस का परीक्षण करने के लिए केवल आवश्यक या पर्याप्त शर्तें ही उपलब्ध हैं। सिस्टम संरचना से घनिष्ठ रूप से बंधे होने के कारण फ्लैट आउटपुट की तलाश में भौतिक पैरामीटर का फायदा उठाने की आवश्यकता होती है। इस कार्य में हम जिस मौलिक प्रश्न का उत्तर देने का प्रयास करते हैं, वह है: एक कीनेमेटिक श्रृंखला में कौन से यांत्रिक तत्वों को बदला या जोड़ा जा सकता है ताकि परिणाम अंडर–एक्ट्यूएटेड एवं फ्लैट हो। इन संशोधनों में सिस्टम मास़–संबंधित पैरामीटर से लेकर काउंटर–बैलेंसिंग मास या स्प्रिंग और डैम्पर्स जैसे निष्क्रिय यांत्रिक तत्वों को जोड़ना शामिल है।

यह कार्य 2–D अंतरिक्ष में रिजिड बॉडीज़ की फ्लैटनेस रिपोर्ट करता है और यह स्थापित करता है कि जटिल कीनेमेटिक श्रृंखलाओं की फ्लैटनेस को अक्सर इसकी छोटी उप–इकाइयों की फ्लैटनेस की पुनरावर्ती परीक्षा से अनुमान लगाया जा सकता है। इस प्रक्रिया में हमें यांत्रिक प्रणालियों की फ्लैटनेस मांगों की एक नई समझ मिलती है। अधिकांश यांत्रिक प्रणालियों को या तो कॉन्फ़िगरेशन–फ्लैट दिखाया गया है या वेग के पहले व्युत्पन्न पर निर्भरता दिखाया गया है। हमने नए कंट्रोल एफाइन सिस्टम का प्रस्ताव दिया है जिसमें टॉर्क और फोर्स फ्लैट आउटपुट के समूह में प्रवेश करते हैं।

बंद श्रृंखला यंत्र–रचना उच्च गति एवं कम इनर्शिया वाले अनुप्रयोगों के लिए उपयुक्त उम्मीदवार हैं। इस थिसिस में हम द्विचर क्लोज्ड–चेन डिफरेंशियल फ्लैट सिस्टम की श्रेणियों का पता लगाते हैं और उन्हें फ्लैट करने के लिए एक व्यवस्थित दृष्टिकोण का प्रस्ताव करते हैं। हम प्रदर्शित करते हैं कि द्रव्यमान के वितरण को ध्यान से चुनकर और स्प्रिंग्स जैसे निष्क्रिय यांत्रिक तत्वों को जोड़कर, एक अंडर–एक्ट्यूएटेड क्लोज–लूप सिस्टम को फ्लैटनेस प्रदान किया जा सकता है। ऐसी प्रणालियों के लिए प्रक्षेपवक्र योजना की प्रभावशीलता

को सिमुलेशन द्वारा प्रदर्शित किया गया है।

सबसे दिलचस्प बात यह है कि डिफरेंशियल फ्लैटनेस उन प्रणालियों में प्रक्षेपवक्र प्रदान करने में सक्षम है जो फ्लैट नहीं हैं, लेकिन एक फ्लैट सन्निकटन की अनुमति देते हैं। हम यांत्रिक प्रणालियों के लिए डिफरेंशियल फ्लैटनेस का एक कमजोर रूप पेश करते हैं जिसे कुछ साहित्य में 'पार्शियल डिफरेंशियल फ्लैटनेस' कहा गया है। अधिकतर, साइक्लिक या इग्नोरेबल निर्देशांक वाले सिस्टम ऐसे गुण प्रदर्शित करते हैं। इस श्रेणी के तहत फ्लोटिंग चेन और सांप जैसे रोबोट की डिफरेंशियल फ्लैटनेस भी स्थापित की गई है।

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NOTATIONS

Notational rules:

- *italic* Roman/Greek lower-case letters refer to scalars.
- *BOLDFACE italic* Roman /Greek upper-case letters with/without subscripts refer to points in a mechanism unless mentioned otherwise.
- **boldface** Roman/Greek lower-case letters refer to vectors.
- **BOLDFACE** Roman /Greek upper-case letters denote matrices.

Letters	Description
θ_i	Angular displacement of i^{th} joint
$\boldsymbol{\theta}$	Vector of joint variables
$\mathbf{I}$	Generalized Inertia matrix
$\boldsymbol{\gamma}$	Vector of gravitational torques
$\mathbf{h}$	Vector of centrifugal and Coriolis forces
$\mathbf{J}$	Jacobian matrix
$\boldsymbol{\lambda}$	Vector of Larange multipliers
g	Acceleration due to gravity $\simeq 9.81\ m/s^2$
l_i	Length of i^{th} body
m_i	Mass of i^{th} body
I_i	Moment of Inertia of i^{th} body about center-of-mass
$\mathbf{x}$	State vector
$\boldsymbol{\tau}$	Generalized force vector
$\mathbf{q}$	Vector of generalized coordinates
q_i	i^{th} generalized coordinate
$\mathbf{F}$	Force vector
f_i	Force component

$\mathbf{f}$	Drift vector
$\mathbf{g}$	Control vector
u_i	Control input at i^{th} joint
t	Time
∇	Directional derivative
$\mathcal{C}$	Accessibility algebra
$C(\mathbf{x})$	Accessibility distribution of Lie brackets
D_i	Lie bracket distribution for feedback linearization
$\mathbb{R}^n$	n-dimensional Real coordinate space
$\mathcal{M}$	Manifold
$\mathbf{y}$	Flat output vector
$\mathbb{C}^n$	Continuity to n^{th} derivative
X, Y and Z	Axes of Cartesian coordinate frame
G_i	Center-of-mass of i^{th} body

ABBREVIATIONS

BVP	Boundary Value Problem
CoM	Center of Mass
CoO	Center of Oscillation
DAC	Differential Algebraic Equation
DeNOC	Decoupled Natural Orthogonal Complement
DoF	Degrees-of-Freedom
EL	Euler-Lagrange
FFSR	Free Floating Space Robot
GJM	Generalized Jacobian Matrix
LARC	Lie Algebra Rank Condition
NE	Newton-Euler
ODE	Ordinary Differential Equation
PB	Primary Body
PBC	Passivity Based Controller
PDF	Partial Differential Flatness
SFL	Static Feedback Linearization
SMC	Sliding Mode Controller
STLC	Small Time Local Controllability
STLCC	Small Time Local Configuration Controllability