

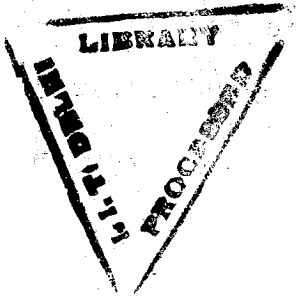
**ANALYSIS OF VIBRATIONS AND THEIR  
EFFECTS IN MATCHED FILTERING**

*By*  
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## PREFACE

Several methods of studying and analyzing vibrations and other motions have recently been developed. Some of these involve classical interferometry, diffraction, moiré, laser speckling and holographic techniques. Out of these, holographic techniques appear to be most informative, as these permit one to study strongly curved surfaces and to visualise complete amplitude distribution over extended areas. In the present thesis we have considered the following problems.

- (i) Holographic study of periodic, non-sinusoidal vibrations and quadratic motion.
- (ii) Analysis of periodic, non-sinusoidal vibrations and quadratic motion using laser speckle interferometry.
- (iii) Effects of vibrations of spatial filter in signal detection experiments.
- (iv) Performance of aerial photographic system in the presence of parabolic motion.
- (v) Effects of asymmetric surface errors of plates on the measurement of vibrations by Fabry-Perot interferometer.

The work reported in the thesis is divided in to seven chapters.

The first chapter is essentially an introduction and briefly reviews earlier work on vibration and motion analysis using optical means. Vibration analysis by holographic

technique is emphasised and compared with other optical methods. Their relative merits and demerits are outlined. An introduction to signal detection by matched filtering technique is also included and effects of static displacement of spatial filter on the performance discussed. The contents of the thesis are also summarized in the present chapter.

It is well known that even linear systems rarely execute simple harmonic motion. So importance of the study of non-sinusoidal vibrations becomes quite evident. In the second chapter, few typical periodic non-sinusoidal vibrations which occur in various realistic situations have been studied holographically. These vibrations are represented by Jacobian elliptic functions  $sn$ -,  $cn$ - and their squares. The characteristic fringe function in some of these cases have been evaluated analytically. Use has been made of an equation derived from considerations of the effect of motion on coherence. The fringe intensity distribution in the reconstructed interferogram in the respective cases are shown graphically. It is noted that in the case of  $sn$ - vibrations intensity of the fringes increases as compared to pure sinusoidal case which results in an increase in the measurement range. In the case of  $cn$ - vibrations the measurement range decreases due to decrease in the intensity of the fringes. Moreover, in the case of  $sn^2$ - or  $cn^2$ - vibrations, the measurement range is not affected with the departure from

pure  $\sin^2$ - or  $\cos^2$ - vibrations.

The direct use of conventional time-average hologram interferometry to investigate vibrations considered in the second chapter is not adequate when the vibrations are of very severe nature. In the third chapter, three techniques namely, holographic addition, holographic subtraction and extended pulse stroboscopic holography have been applied to study these severe periodic, non-sinusoidal vibrations in detail. It is, however, noted that application of these techniques extends the amplitude measurement range of conventional time-average hologram interferometry. The techniques of holographic addition and subtraction have also been applied to study quadratic motion of severe nature and an extension in the measurement range is reported.

Laser speckle interferometry is a well known technique to analyse and measure motion and vibrations of diffusely reflecting objects. In the fourth chapter, a general expression has been derived giving contrast of the laser speckles as a function of ratio of reference to object beam intensities and the characteristic fringe function of motion. Resulting speckle fringes in the case of periodic, non-sinusoidal vibrations ( represented by Jacobian elliptic functions  $sn$  ,  $cn$  and their squares ) and quadratic motion are shown graphically. It is observed that the speckle visibility with

a high reference to object beam intensity ratio tends to follow the fringes observed in conventional time-average hologram interferometry. A low reference to object beam intensity ratio, however, results in low contrast of fringes.

Detection of known signal within a noisy background by matched filtering is probably the most commonly practised coherent optical data processing operation of the present time. In this technique use is made of a complex spatial filter generated holographically. In the fifth chapter, the effects of transverse and longitudinal vibrations of the spatial filter on the performance of the matched filtering system has been investigated. It is noticed that in both the cases, the performance of the system is degraded which is illustrated graphically against amplitude of vibrations. However, it is observed that the effect of transverse vibrations is more severe than that of longitudinal vibrations. Effects of two types of periodic, non-sinusoidal vibrations, represented by Jacobian elliptic functions  $sn$ - and  $cn$ - , have also been investigated. It is noticed that in the case of vibrations represented by  $sn$ -function, the decrease in performance is more rapid than in the case of pure sinusoidal vibrations, whereas, in the case of vibrations represented by  $cn$ - functions, an improvement in the performance of the system has been observed.

In aerial photography, it is important to know the influence of image motion on the performance of photographic systems. In the sixth chapter, this problem has been considered using Schell's theorem in the frame work of the theory of partial coherence. The performance characteristics of an aberration free optical system with circular aperture , operating in partially coherent light in presence of parabolic motion have been evaluated. The results are illustrated graphically for various values of motion parameter and different types of coherence functions which occur in realistic situations . Results have also been compared with the case of linear motion.

Two-beam and multiple-beam interferometers are widely used to measure mechanical vibrations. However, the performance of a multiple-beam interferometer is severely limited by the lack of perfect planeness and parallelism between the plates. In the seventh chapter, the effects of asymmetric surface errors of plates on the measurement of vibration amplitudes by a Fabry-Perot interferometer have been discussed. The distribution of irradiance in the fringes of equal inclination in the presence of sinusoidal vibrations and asymmetric surface defects in one of the plates is illustrated graphically. Resulting errors in the measurement of vibration amplitude have been point out. Results have also been compared with the case of symmetric errors.

The application of the technique of holographic weighted subtraction to estimate amplitudes of  $sn-$ ,  $cn-$ ,  $sn^2-$  and  $cn^2-$  vibrations quantitatively has been reported in appendix A. It has been shown that in the case of  $sn-$  and  $cn-$  vibrations, this technique can be used to generate contour lines of equal vibration amplitudes on the object at any given small level. The higher vibration amplitudes can also be studied easily using this technique. In the case of  $sn^2-$  and  $cn^2-$  vibrations, the technique, however, does not offer any additional flexibility in the study of small amplitudes.

The analysis of  $sn-$ ,  $cn-$ ,  $sn^2-$  and  $cn^2-$  vibrations, when thin phase material is used for time-average hologram recording, is included in appendix B. It has been shown that use of such material as recording media in motion analysis makes the higher order fringes more intense. In the case of  $sn-$  vibrations the fringe structure becomes complicated and in the case of  $cn-$  vibrations it is simplified as compared with pure sinusoidal vibrations whereas, in the case of  $sn^2-$  or  $cn^2-$  vibrations, the entire fringe structure changes as compared with the pure  $\sin^2-$  or  $\cos^2-$  vibrations.

A part of the work reported in this thesis has resulted in the following research articles.

- (1) P.C. Gupta and K. Singh ; Appl. Opt. 14(1975) 129-33  
Characteristic Fringe Function for Time-Average  
Holography of Periodic, Non-Sinusoidal Vibrations.
- (2) P.C. Gupta and K. Singh ; Appl. Phys. 6 (1975) 233-41  
Time-Average Hologram Interferometry of Periodic,  
Non-Cosinusoidal Vibrations.
- (3) P.C. Gupta and K. Singh; J. Optics **3(1974) 43-45.**  
Study of Typical Periodic, Non-Sinusoidal Vibrations  
by Time-Average Hologram Interferometry.
- (4) P.C. Gupta and K. Singh; Opto-Electronics 6 (1974) 305-11  
Holographic Interferometry of Non-Sinusoidal  
Vibrations.
- (5) P.C. Gupta and K. Singh; Nouv. Rev. d'Opt. (In Press)  
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- (6) P.C. Gupta and K. Singh; Pramana 3 (1974) 390-96  
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to Quadratic Motion Analysis.
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- (8) P.C. Gupta and K. Singh; Appl. Phys. (In Press)  
Effects of Vibrations of Spatial Filter in Signal  
Detection by Matched Filtering.

- (9) K.Singh and P.C. Gupta; Appl.Opt. (In Press).  
Effects of Periodic, Non-Sinusoidal Vibrations in Matched Filtering for Detection of Signals Buried in Noise of Uniform Spectral Density.
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- (11) P.C. Gupta and K. Singh; Appl.Opt. (In Press)  
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Vibration Measurement by a Fabry-Perot Interferometer with Asymmetric Surface Errors on Plates.
- (13) P.C. Gupta and K. Singh ; Indian J.Phys. (submitted)  
Holographic Weighted Subtraction in Time-Average Hologram Interferometry of Periodic, Non-Sinusoidal Vibrations.
- (14) P.C. Gupta and K. Singh ; Wave-Electronics (In Press)  
Time-Average Hologram Interferometry of Periodic, Non-Sinusoidal Vibrations with Thin Phase Recording Materials.
- (15) P.C. Gupta and K. Singh; in "Symposium On Lasers", Indian Institute of Technology, Kanpur (August 14-16,1975)  
Laser Speckle Interferometry of Constant Velocity and Quadratic Motions.

In addition to the work reported in the thesis, the author has published following papers.

- (i) K.Singh, P.C. Gupta and R.Rattan; Att.Fond.G.Ronchi  
28(1973)851-55.  
Multiple-Beam Interferometric Method of Studying Small Vibrations.
- (ii) R.Rattan, P.C. Gupta and K.Singh; Proc.Ind.Natn.Sci.  
Acad.34 (1973) 320-27.  
Effects of Surface Errors and Misalignment of Plates on the Vibration Measurement by a Fabry-Perot Interferometer.
- (iii) R.Rattan, P.C. Gupta and K.Singh Att.Fond.G.Ronchi  
26 (1972) 849-65.  
Partially Coherent Imagery in the Presence of Transverse Vibrations and Linear Image Motion.
- (iv) P.C. Gupta and K. Singh in "All India Workshop and Symposium on Digital Image Processing", Indian Institute of Sciences, Bangalore (Nov.26 - Dec. 1, 1973)  
Coherent Optical Filtering:Introduction and Applications.
- (v) A.K. Agarwal and P.C. Gupta; in "Symposium on Lasers" ,  
Indian Institute of Technology, Kanpur (August 14-16,1975)  
Effect of Filter Recording Errors in Optical Signal Detection.

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