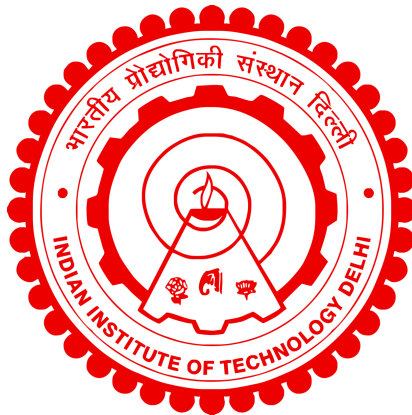


**COMPUTATIONAL STUDY OF TURBULENT
FORCED PLUME IN A STRATIFIED MEDIUM
SUBJECTED TO VOLUMETRIC HEATING**

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DEPARTMENT OF APPLIED MECHANICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

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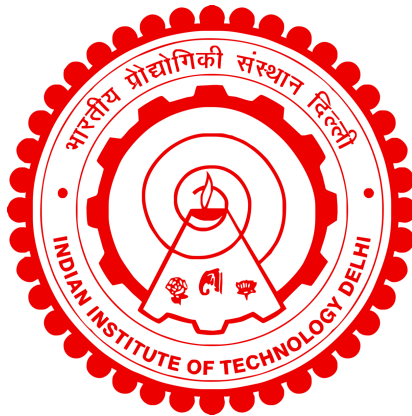
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Certificate

This is to certify that the thesis entitled “**Computational study of turbulent forced plume in a stratified medium subjected to volumetric heating**”, submitted by **Mr. Nitin Kumar** to the **Indian Institute of Technology Delhi** for the award of the degree of **Doctor of Philosophy** is a record of the original, bona fide research work carried out by him under our supervision. The thesis works meets the requisite standards and the candidate is worthy of consideration for the degree of Doctor of Philosophy in accordance with the regulations of the institute.

The results contained in this thesis have not been submitted in part or in full to any other university or institute for the award of any degree or diploma.

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Abstract

Turbulent plumes are commonly encountered in both natural and engineering flows. One of their most significant applications is in atmospheric flows, where clouds are modeled as a combination of multiple plumes evolving in a stratified environment. As the moist air parcel rises up, it becomes saturated and the water vapor condenses, leading to the release of latent heat. This additional buoyancy due to latent heat release in clouds is typically modeled in the laboratory as the volumetric heating of plumes. Atmospheric stability also plays a critical role in the evolution of clouds as it determines the rate at which the density of the atmosphere varies in the vertical direction. Cloud convection typically occurs within a stably stratified environment, which consists of layers of vertically varying densities in the atmosphere. In this study, we investigate the behavior of volumetrically heated plumes in a stratified medium using both RANS modeling and large eddy simulations. The evolution of mean and turbulent statistics of heated plumes are examined in detail.

We first conducted numerical simulations of the forced plume in a stratified medium using the Reynolds-Averaged Navier-Stokes (RANS) equations paired with the standard k - ϵ turbulence model. This approach allowed us to perform many simulations at much lower computational cost when compared to large eddy simulations. We systematically varied the background stratification and examined how the flow statistics vary with the stratification. This also allowed us to validate the RANS simulations against the reported experimental data [1]. We compared the maximum height, mean centerline velocity, and turbulence statistics such as shear production and dissipation rate. Mean velocity profiles obtained from RANS simulations showed good agreement with the experimental data when the background is unstratified i.e. when the density of the ambient is uniform. However, mean flow statistics for a stratified medium showed considerable deviation between the RANS simulations and experimental data. We found that as the stratification strength increased, the maximum height of the plume decreased. This suggests the importance of atmospheric stability in limiting the vertical development of clouds. Further, we examined how the turbulence energy budget varies with the background stratification strength. We noted that for stronger stratification levels, the residual of the turbulence kinetic energy budget increased. This suggests a greater non-homogeneity in the flow,

indicating the limitations of the assumptions made in RANS models. Overall, the RANS modeling was effective in studying the mean flow quantities for weakly stratified and unstratified mediums, however, at higher stratification strengths significant improvements in the modeling are required.

In the second part of this study, we performed large eddy simulations (LES) of the forced plume subjected to volumetric heating in a stratified medium. The heating is computationally modeled by adding a source term to the energy equation near the plume equilibrium height. We validated our LES methodology by comparing the mean flow statistics and the entrainment rate coefficient with previous experimental study [2], for an unstratified medium. We performed a systematic study by varying two key parameters in the evolution of heated plumes. First is the heating rate which determines the rate at which energy is added to the system due to latent heat release, and the second parameter is the background stratification which determines the rate at which the background density varies in the vertical direction. In our study, we considered a similar range of heating rates as observed in cumulus clouds. We found that the addition of heat accelerated the plume due to additional buoyancy leading to an increase in the mean axial velocity. Mean velocity profiles were found to preserve their Gaussian structure in the heating zone, however, the temperature profile shows a double hump structure. At the same time, the turbulent kinetic energy and vorticity magnitude decreased in the heating zone due to the disruption of coherent structures caused by heating. The entrainment rate increases at the start of the heat injection zone (HIZ), but then drastically reduces towards the end of the HIZ. We observed that as the plume rises above the heating zone, it restores its Gaussian shape for temperature. Also, the vorticity magnitude and turbulence kinetic energy (TKE) increased above the heating zone. Interestingly, the kinetic energy spectrum reveals different scaling behaviors: κ_z^{-3} for an unheated plume and $\kappa_z^{-11/5}$ for a heated plume.

On the other end, increasing the background stratification is found to counteract the effects of volumetric heating by suppressing the maximum plume height and turbulence levels. The entrainment coefficient is observed to decrease with an increase in stratification strength for a given heating rate. Primary difference between the stratified and unstratified mediums is that in the unstratified medium where the density of the ambient is uniform, plumes rise asymptotically without any maximum

height. However, in a stratified medium where the plume encounters continuously varying density layers, the density of the plume at a certain height matches with the ambient, and the plume starts to spread laterally with the maximum height of the plume slightly above the neutral layer. In the case of a heated plume, due to additional buoyancy provided by the heating, the plume has a new neutral layer and reaches a new maximum height above the heating zone.

Next, we investigated the entrainment dynamics of a forced plume with volumetric heating using energy-consistent entrainment relations. We decomposed the entrainment coefficient into different components for both unheated and heated plumes. The closure of the energy-consistent approach is compared to the total entrainment coefficient with the standard entrainment hypothesis.

According to the energy consistent entrainment relations, there are three contributors to the entrainment coefficient, 1) contribution from the turbulence energy production, 2) contribution from the buoyancy effects, 3) contribution due to departure from self-similarity. In the unstratified medium, turbulence energy production is found to be the primary driver of the entrainment in unheated plumes, while for the heated plumes, the contribution from the turbulence energy production reduced, and the buoyancy emerged as the dominant factor driving the entrainment within the HIZ and beyond. In the stratified medium, the entrainment coefficient for the unheated plume due to the buoyancy effects is negative in the neutral layer. However, the release of latent heat leads to additional buoyancy, thus the contribution of buoyancy to the entrainment coefficient becomes less negative. As a result, the cumulative entrainment coefficient goes from a negative value (detrainment zone) to a positive value (entrainment zone) towards the end of HIZ. As the stratification strength increases, the contribution from the turbulent production to the entrainment coefficient increases. We also found that the deviation from expected self-similar behavior was more pronounced as the stratification strength increased.

सारांश

टर्बुलेंट प्लूम प्राकृतिक और अभियांत्रिकी प्रवाहों में सामान्य रूप से पाए जाते हैं। इनका सबसे महत्वपूर्ण अनुप्रयोग वायुमंडलीय प्रवाहों में है, जहाँ बादलों को कई प्लूम के संयोजन के रूप में मॉडल किया जाता है जो एक स्तरीकृत वातावरण में विकसित होते हैं। जैसे-जैसे नम वायु कण ऊपर उठता है, वह संतृप्त हो जाता है और जल वाष्प संघनित हो जाता है, जिससे अव्यक्त ऊष्मा मुक्त होती है। इस अतिरिक्त उत्प्लावकता को प्रयोगशाला में आमतौर पर प्लूम के आयतनिक तापन के रूप में मॉडल किया जाता है। वायुमंडलीय स्थिरता बादलों के विकास में महत्वपूर्ण भूमिका निभाती है क्योंकि यह निर्धारित करती है कि वायुमंडल में घनत्व ऊर्ध्वाधर दिशा में कैसे बदलता है। बादल संवहन आमतौर पर एक स्थिर स्तरीकृत वातावरण में होता है, जिसमें वायुमंडल में ऊर्ध्वाधर रूप से बदलते घनत्व की परतें होती हैं। इस अध्ययन में, हमने एक स्तरीकृत माध्यम में आयतनिक रूप से तापित प्लूम के व्यवहार की जांच की, जिसमें रेनॉल्ड्स-औसत नेवियर-स्टोक्स समीकरणों (रैन्स) और बड़े एडी सिमुलेशन दोनों का उपयोग किया गया।

सर्वप्रथम, हमने एक स्तरीकृत माध्यम में बलपूर्वक प्लूम की संख्यात्मक सिमुलेशन मानक $k-\epsilon$ टर्बुलेंस मॉडल के साथ रेनॉल्ड्स-औसत नेवियर-स्टोक्स समीकरणों का उपयोग करके की। यह दृष्टिकोण बड़े एडी सिमुलेशन की तुलना में बहुत कम कम्प्यूटेशनल लागत पर कई सिमुलेशन करने की अनुमति देता है। हमने पृष्ठभूमि स्तरीकरण को क्रमबद्ध रूप से परिवर्तित किया और जांच की कि कैसे प्लूम के आंकड़े स्तरीकरण के साथ बदलते हैं। इसने हमें प्रयोगात्मक डेटा के साथ रैन्स सिमुलेशन को मान्य करने में सहायता की। हमने अधिकतम ऊंचाई, औसत केंद्र रेखा वेग और टर्बुलेंस आंकड़ों जैसे अपरदन उत्पादन और विलोपन दर की तुलना की। रैन्स सिमुलेशन से प्राप्त औसत वेग प्रोफाइल प्रयोगात्मक डेटा के साथ अच्छी सहमति दिखाते हैं जब पृष्ठभूमि अस्तरीकृत होती है, यानी जब परिवेश का घनत्व समान होता है। हालांकि, स्तरीकृत माध्यम के लिए औसत प्लूम आंकड़ों में रैन्स सिमुलेशन और प्रयोगात्मक डेटा के बीच काफी विचलन पाया गया। हमने पाया कि जैसे-जैसे स्तरीकरण की शक्ति बढ़ी, प्लूम की अधिकतम ऊंचाई घटती गई। यह वायुमंडलीय स्थिरता की महत्वपूर्ण भूमिका को दर्शाता है जो बादलों के ऊर्ध्वाधर विकास को सीमित करती है।

हमने यह भी जांचा कि पृष्ठभूमि स्तरीकरण की शक्ति के साथ टर्बुलेंस ऊर्जा बजट कैसे बदलता है। हमने देखा कि अधिक मजबूत स्तरीकरण स्तरों के लिए, टर्बुलेंस गतिज ऊर्जा बजट का अवशेष बढ़ गया। यह प्लूम में अधिक असमानता का संकेत देता है, जो रेन्स मॉडल में किए गए मान्यताओं की सीमाओं को दर्शाता है। कुल मिलाकर, रेन्स मॉडलिंग कमजोर स्तरीकृत और अस्तरीकृत माध्यमों में औसत प्लूम मात्राओं का अध्ययन करने में प्रभावी थी, हालांकि उच्च स्तरीकरण शक्तियों पर मॉडलिंग में महत्वपूर्ण सुधार की आवश्यकता है।

दूसरे भाग में, हमने एक स्तरीकृत माध्यम में आयतनिक तापन के अधीन बलपूर्वक प्लूम के लिए बड़े एडी सिमुलेशन किए। तापन को संगणनात्मक रूप से प्लूम की संतुलन ऊंचाई के निकट ऊर्जा समीकरण में एक स्रोत पद जोड़कर मॉडल किया गया। हमने अस्तरीकृत माध्यम के लिए पूर्व प्रयोगात्मक अध्ययन के साथ औसत प्लूम आंकड़ों और प्रवेशन दर गुणांक की तुलना करके अपनी बड़े एडी सिमुलेशन पद्धति को मान्य किया। हमने दो महत्वपूर्ण मापदंडों को बदलकर एक व्यवस्थित अध्ययन किया। पहला तापन दर है जो अव्यक्त ऊष्मा के कारण प्रणाली में ऊर्जा जोड़ने की दर को निर्धारित करता है, और दूसरा पृष्ठभूमि स्तरीकरण है जो ऊर्ध्वाधर दिशा में पृष्ठभूमि घनत्व बदलने की दर को निर्धारित करता है। हमने संचयी बादलों में देखी गई तापन दरों की समान श्रेणी पर विचार किया। हमने पाया कि ऊष्मा की अतिरिक्त उत्प्लावकता के कारण प्लूम की गति बढ़ गई। औसत वेग प्रोफाइल तापन क्षेत्र में गाउसियन संरचना को बनाए रखती हैं, हालांकि तापमान प्रोफाइल में दोहरी उभार संरचना दिखाई देती है। साथ ही, तापन के कारण सुसंगत संरचनाओं में भंग होने के कारण टर्बुलेंस गतिज ऊर्जा और भ्रंश परिमाण कम हो गया। प्रवेशन दर तापन प्रवेश क्षेत्र (HIZ) के शुरुआती भाग में बढ़ती है, लेकिन फिर HIZ के अंत की ओर तेजी से कम हो जाती है। हमने देखा कि जब प्लूम तापन क्षेत्र से ऊपर उठता है, तो यह अपना गाउसियन आकार पुनः प्राप्त करता है। भ्रंश परिमाण और टर्बुलेंस गतिज ऊर्जा तापन क्षेत्र के ऊपर बढ़ती है। दिलचस्प बात यह है कि गतिज ऊर्जा स्पेक्ट्रम में अलग-अलग मापक व्यवहार दिखाई देते हैं: बिना तापे गए प्लूम के लिए κ_z की माप -3 और तापित प्लूम के लिए κ_z की माप -11/5।

दूसरी ओर, पृष्ठभूमि स्तरीकरण में वृद्धि को तापन के प्रभावों को काउंटर करते हुए पाया गया, जिससे अधिकतम प्लूम ऊंचाई और टर्बुलेंस स्तर दमित हो गए। किसी दिए गए

तापन दर के लिए स्तरीकरण की शक्ति में वृद्धि के साथ प्रवेशन गुणांक में कमी देखी गई। अस्तरीकृत और स्तरीकृत माध्यमों के बीच मुख्य अंतर यह है कि अस्तरीकृत माध्यम में जहां परिवेश का घनत्व समान होता है, प्लूम बिना किसी अधिकतम ऊंचाई के असिंप्टोटिक रूप से ऊपर उठता है। जबकि स्तरीकृत माध्यम में, जहां प्लूम निरंतर बदलते घनत्व की परतों का सामना करता है, प्लूम का घनत्व एक निश्चित ऊंचाई पर परिवेश के घनत्व के बराबर हो जाता है, और प्लूम पार्श्विक रूप से फैलने लगता है, प्लूम की अधिकतम ऊंचाई तटस्थ परत से थोड़ा ऊपर होती है। तापित प्लूम के मामले में, तापन द्वारा प्रदान की गई अतिरिक्त उत्प्लावकता के कारण प्लूम का एक नया तटस्थ परत होता है और तापन क्षेत्र से ऊपर एक नई अधिकतम ऊंचाई तक पहुंचता है।

इसके बाद, हमने ऊर्जा-संगत प्रवेशन संबंधों का उपयोग करके एक बलपूर्वक प्लूम के प्रवेशन गतिकी की जांच की। हमने बिना तापे गए और तापित प्लूम के लिए प्रवेशन गुणांक को विभिन्न घटकों में विभाजित किया। ऊर्जा-संगत प्रवेशन संबंधों के अनुसार, प्रवेशन गुणांक में तीन योगदानकर्ता होते हैं: 1) टर्बुलेंस ऊर्जा उत्पादन से योगदान, 2) उत्प्लावकता प्रभावों से योगदान, 3) स्व-समानता से विचलन के कारण योगदान।

अस्तरीकृत माध्यम में, बिना तापे गए प्लूम में टर्बुलेंस ऊर्जा उत्पादन प्रवेशन का प्राथमिक चालक पाया गया, जबकि तापित प्लूम में, टर्बुलेंस ऊर्जा उत्पादन का योगदान कम हो गया और उत्प्लावकता तापन प्रवेश क्षेत्र और उसके बाहर प्रवेशन का प्रमुख कारक बन गई। स्तरीकृत माध्यम में, बिना तापे गए प्लूम के लिए उत्प्लावकता प्रभावों के कारण प्रवेशन गुणांक तटस्थ परत में नकारात्मक होता है। हालांकि, अव्यक्त ऊष्मा के मुक्त होने से अतिरिक्त उत्प्लावकता मिलती है, इसलिए प्रवेशन गुणांक में उत्प्लावकता का योगदान कम नकारात्मक हो जाता है। परिणामस्वरूप, संचयी प्रवेशन गुणांक एक नकारात्मक मान (विप्रवेशन क्षेत्र) से एक सकारात्मक मान (प्रवेशन क्षेत्र) की ओर बढ़ता है तापन प्रवेश क्षेत्र के अंत की ओर। जैसे-जैसे स्तरीकरण की शक्ति बढ़ती है, प्रवेशन गुणांक में टर्बुलेंट उत्पादन का योगदान बढ़ता जाता है। हमने यह भी पाया कि अपेक्षित स्व-समान व्यवहार से विचलन स्तरीकरण की शक्ति में वृद्धि के साथ अधिक स्पष्ट हो गया।

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Abbreviations

RANS	R eynolds- A veraged N avier S tokes
LES	L arge E ddy S imulation
DNS	D irect N umerical S imulations
OpenFOAM	O pen F ield O peration A nd M anipulation
CFD	C omputational F luid D ynamics
TKE	T urbulent K inetic E nergy
PISO	P ressure I mplicit S plitting O perator
SIMPLE	S emi I mplicit M ethod F or P ressure L inked E quation
HIZ	H eat I njection Z one

Symbols

Γ	Flux-balance parameter
α	Coefficient of entrainment
g	Acceleration due to gravity
ρ	Fluid density
ρ_0	Reference density
x	Coordinate in x-direction
y	Coordinate in y-direction
z	Vertical coordinate
u	Velocity in x-direction
v	Velocity in y-direction
w	Vertical velocity
T	Instantaneous Temperature
T_a	Ambient Temperature
T_d	Deviation Temperature from ambient
T_b	Reference temperature
p	Instantaneous Pressure
p_d	Deviation Pressure
D	Diameter of the source
t	Time
t^*	Non-dimensional time
ν	Kinematic viscosity
κ	Diffusivity
β	thermal expansion coefficient
Re	Reynolds number
Ri	Richardson number
B_0	Source buoyancy flux

M_0	Source momentum flux
Pr	Prandtl number
N_∞	Buoyancy frequency
Z_m	Maximum height
Z_s	Spreading height
G	Heat release number
ν_{sgs}	Sub-grid scale viscosity
Q	Volume flux
M	Momentum flux
F	Buoyancy flux
Δ	Filter width
L	Length of heat injection zone
g'	Reduced gravity
z_b	Vertical distance of the HIZ from the source