ALGORITHMIC STUDIES IN GRAPH CONNECTIVITY AND IN NETWORK FLOWS

by

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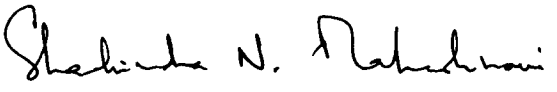
CERTIFICATE

This is to certify that the thesis entitled "Algorithmic Studies in Graph Connectivity and in Network Flows" being submitted by J.Cheriyan to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy is a record of bonafide research work carried out under my supervision. The results contained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.



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ABSTRACT

Two spanning trees of a graph are independent if they are rooted at the same vertex r and for each remaining vertex v the two paths from v to r, one path in each tree, are internally vertex disjoint. Further, an edge incident with r is contained in at most one spanning tree. k spanning trees are independent if they are pairwise independent.

The focus of the graph connectivity work in this thesis is on constructing three independent spanning trees in any 3-connected graph. A new characterisation of 3-connected graphs, namely the nonseparating ear decomposition theorem, is developed and using this characterisation an "extended s-t" numbering is assigned to the vertices. This numbering straight away gives three independent spanning trees.

A cycle C of graph G(V,E) is a nonseparating induced cycle if G\V(C) is connected and C has no diagonals. A new linear time dfs algorithm is developed which given an edge tr and vertex u, $t \neq u \neq r$, of any 3-connected graph finds a nonseparating induced cycle through tr and avoiding u. Using this algorithm at most V times it is possible to construct a nonseparating ear decomposition.

Goldberg and Tarjan recently introduced the maximum distance preflow push algorithm for finding a maximum network flow and gave an $O(V^3)$ time bound for their algorithm. Here the bound is improved to $O(V^2\sqrt{E})$ and this bound is shown to be tight by constructing a parametrized worst case network on which the algorithm requires $\Omega(V^2\sqrt{E})$ time. An improved maximum flow algorithm for the synchronous distributed model of computation which uses $O(V^2\sqrt{E})$ messages and $O(V^2)$ time is also developed.

The blocking flow problem is to find a flow such that each source to sink path in the given network contains a saturated edge. A 3-layer network is an acyclic network in which all source to sink paths have length three. It is shown that the problem of finding the lexicographically first blocking flow in a 3-layer network is P-complete; however, a random NC algorithm for finding an arbitrary blocking flow in a 3-layer network is given.

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