FUZZY MATRIX AND BI-MATRIX GAMES

by

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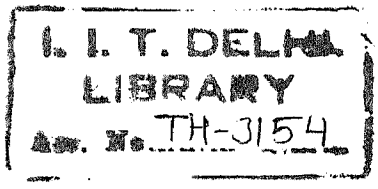

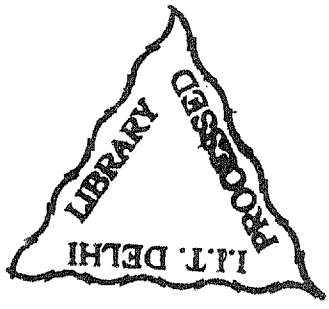

Certificate

This is to certify that the thesis entitled **Fuzzy Matrix and Bi-matrix Games**, which is being submitted by **Vidyottama Vijay** for the award of the degree of **Doctor of Philosophy in Mathematics** to the **Indian Institute of Technology Delhi**, is a bona fide research work done under our guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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Abstract

The thesis aims at conceptualizing the two person matrix game theory in fuzzy environment and at providing the relationship between such a fuzzy game and fuzzy optimization problems.

The thesis comprises of five chapters. The first chapter is introductory in nature and pertains to a brief review of the related work to the proposed study besides containing the summary of the research work presented in this thesis.

The purpose of Chapter 2 is to conceptualize a matrix game with fuzzy goals and to show its equivalence with a primal-dual pair of fuzzy linear programming problems. Further, Nishizaki and Sakwa's model for the matrix game with fuzzy goals is also presented along with its similarities and differences with the present model for the same.

In Chapter 3, duality theory for linear programming problems with fuzzy parameters is established by employing a defuzzification function approach. Further, it is shown that a matrix game with fuzzy payoffs is equivalent to a primal-dual pair of such fuzzy linear programming problems. Also, certain difficulties with similar studies reported in the literature are discussed. Moreover, the solution to a matrix game with fuzzy goals and fuzzy payoffs is conceptualized by using a suitable defuzzification function and it is proved that such a fuzzy game is equivalent to a primal-dual pair of fuzzy linear programming problems.

Chapter 4 is concerned with bi-matrix games in fuzzy environment. For bi-matrix games with fuzzy goals, a Zimmermann type approach is followed, while a defuzzification function approach is suggested for bi-matrix games with fuzzy payoffs. These two approaches are then unified to study bi-matrix games with fuzzy goals and fuzzy payoffs. In each of these cases, it is shown that the given fuzzy bi-matrix game is equivalent to a suitable (crisp) non-linear programming problem in which the objective as well as all constraint functions are linear except two constraint functions which are quadratic.

The last chapter of the thesis, Chapter 5, is devoted to the model for the matrix game with fuzzy goals and fuzzy payoffs developed by using fuzzy relation approach. Further, it has been shown that such a fuzzy matrix game based on the fuzzy relation approach is equivalent to two optimization problems. In general, duality theory between these fuzzy optimization problems can not be established. However, it has been shown here that for some special cases certain duality results could be obtained.

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